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L. EULERI
OPUSCULORUM
TOMUS III.
CONTINENS
NOVAM THEORIAM MAGNETIS
AB ILLUSTR. ACADEMIA REGIA
SCIENT: PARISINA
PRÆMIO CONDECORATAM A. 1744.

UNA CUM NONNULLIS ALIIS DISSERTATIONIBUS ANALTTICO-MECHANICIS.



BEROLINI,
SUMTIBUS AMBR. HAUDE ET JOH. CAROL. SPENERI,
BIBLIPOL. REG. ET ACAD. SCIENT. PRIVIL.
1751.

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DISSERTATIO DE MAGNETE

ab Illuſtr. Academia Regia Paris. Scient. præmio condecorata.

A. 1744.

§. I.

Tab. I. & II

Virtutis magneticæ cauſa phyſica datur.

Credo ego Illuſtriſſimam Academiam Regiam illorum opinioni minime aſſentiri, qui cauſam virtutis magneticæ quærendo defatigati non dubitant ullam horum phænomenorum cauſam intelligibilem dari, pertinaciter negare. Hoc certe mihi quidem clariſſime confirmare videtur repetita hujus ejusdem quæſtionis propoſitio, qua Academia Regia ſatis luculenter publice declarat, quærendi defatigationem eſſe turpiſſimam, cum id, quod quæritur,

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ſit pulcerrimum. Cum enim Illuſtr. Academia ante biennium hanc de magnete quæſtionem cum lauti præmii promiſſione propoſuiſſet, neque tamen quicquam inveniſſet, in quo acquieſcere poſſet, graviſſimam ſane cauſam hanc quæſtionem deferendi habuiſſe videatur. Verum tantum abeſt, ut iſtam inveſtigationem propterea relinquendam cenſeat, ut potius præmium duplicare, hocque magis animos ad iſtud negotium ſuſcipiendum inflammare decreverit.

Hæc cauſa eſt vehementer abſcondita.

§. II. Quanquam equidem nunquam dubitavi, quin omnes naturæ effectus a cauſis mechanicis proficiſcantur, ac ſemper malui ignorantiam meam palam profiteri, quam quicquam ſine cauſa fieri, dicere; tamen natura magnetis perpetuo mihi tam ardua ac tantopere abſcondita eſt viſa, ut omne ſtudium atque operam in ejus indagatione fruſtra a me impendi ſim arbitratus: quæ cauſa me etiam abſterruit, ut, cum primum hæc quæſtio proponeretur, in ea enodanda nequidem elaborare ſim auſus. Non quo mihi dignitas quæſtionis minus eſſet perſpecta, ſed quoniam vidi, tot præſtantiſſimorum virorum, qui hoc opus ſunt aggreſſi, conatus irritos fuiſſe omnes. Imprimis vero me detinuit Celeb. Muſchenbrockii provocatio, qui conquiſitis atque examinatis ſummo ſtudio experimentis id tandem ſe eviciſſe jactat, virtutum magneticarum cauſam prorſus non eſſe mechanicam, neque ulli ſubſtantiæ materiali adſcribi poſſe. Quod etſi mihi quidem minime perſuaſit, tamen ſpem ademit omnem in hoc ſublimi negotio quicquam proficiendi.

Non tam fons quam explicatio virtutis magneticæ deſideratur.

§. III. Si hoc conſilium protinus ſequi voluiſſem, nunc certe maxime ab hoc onere me abſtinere oporteret, poſtquam tot egregia ſummoque ſtudio elaborata tentamina ipſius Academiæ aequiſſimo judicio ſint repudiata. Quæ cum apud me diligentius evolviſſem, fieri profecto non poſ-

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ſe mihi viſum eſt, ut omnes conjecturæ, quæ ad phænomena magnetis explicanda adhuc ſunt excogitatæ, a veritate aequaliter abhorreant; propterea quod nullum fontem, unde ejusmodi vires promanare queant, prætermiſſum eſſe videbam. Quamobrem non tam in novum fontem, ex quo vires magneticæ deriventur, inquirendum eſſe intellexi, quam in ipſum derivationis modum. Neque idcirco primam originem adhuc latere exiſtimo, ſed ſolum rationem, qua ex jam cognita origine ſingula magnetis phænomena dilucide deduci atque explicari queant. Quod negotium cum ſit difficillimæ indaginis propter plurimas quæſtiones maxime intricatas ante evolvendas, non amplius tantopere miror, evidentem horum phænomenorum explicationem etiamnunc deſiderari.

§. IV. Omnino autem Carteſius mihi videtur, qui iſtam explicationem primus fere cum ratione eſt aggreſſus, non adeo a ſcopo aberraſſe, ut ejus idea penitus rejici debeat. Quamvis enim totum negotium neutiquam confecerit, tamen naturam ſecutus neque in prima cauſa aſſignanda neque in ipſa explicationis ratione tantum a veritate receſſiſſe videtur, quantum factum eſſe comperio a pluribus Philoſophis recentioribus, qui dum ejus ideas reformare atque emendare voluerunt, in craſſiſſimos errores inciderunt. Atque omnino noviſſimi Philoſophi mihi quidem videntur eo longius a veritate digreſſi, quo majus ſtudium attulerunt ad anteceſſorum ſententias evertendas, et explodendas: ſicque novitatis ſtudium cognitioni veritatis vehementer impedimento fuiſſe exiſtimo. Hoc pacto enim factum eſt, ut profligatis omnibus rationalibus modis naturam magnetis explicandi, nil aliud relinqueretur, niſi ut cauſa incorporea in medium produceretur.

Recentiores Phyſici parum in hac explicatione profecerunt.

Cauſa virtutis magneticæ partim intra partim extra magnetem quæri debet.

§. V. Quoniam igitur dubium eſt nullum, quin omnes magnetis effectus a cauſa corporea ſive mechanica proficiſcantur, hanc cauſam partim in interna magnetis ſtructura, partim in materia extra magnetem ſubſiſtente quæri oportebit. Cum enim hæc virtus, cujus cauſam inveſtigamus, non in omnibus corporibus, ſed in certa tantum ſpecie, ad quam ſcilicet magnetem ac ferrum magnetica virtute imbutum referimus, reperiatur, neceſſe eſt ut interna horum corporum indoles ſit prorſus peculiaris, atque a natura reliquorum corporum, in quæ talis vis non cadit, penitus diverſa. Quomodocunque autem hæc interna corporum magneticorum conſtitutio ſit comparata, tamen ab ea ſola oriri non poſſunt illa phænomena, quibus videmus duos magnetes ſe mutuo non contingentes in ſe invicem cum attrahendo, tum repellendo, tum etiam dirigendo agere, et vires exercere. Ex quo perſpicuum eſt præter internam corporum magneticorum ſtructuram adeſſe materiam quandam hæc corpora ambientem, atque extra illa ſubſiſtentem, quæ cum ipſa ſtructura cuncta phænomena producere valeat.

Cauſæ externæ exiſtentia a Philoſophis pluribus in dubium vocari ſolet.

§. VI. Quod ad priorem cauſæ partem attinet, neminem fore arbitror, qui internum magnetis ſtatum a reliquis corporibus diſcrepare neget. Etiamſi enim magnes præ reliquis corporibus non præditus eſſet tam eximiis virtutibus, tamen propter hoc ipſum, quod differat a reliquis corporibus, nemo dubitaret in ipſo ſtructuram peculiarem, qua diſtinguatur, ſtatuere. Longe aliter autem de materia illa externa, in qua alteram cauſæ partem poſitam eſſe dico, nonnullos Philoſophos hoc præſertim tempore ſentire video, qui pertinaciter negant ullam materiam ſubtilem ad naturæ phænomena explicanda adhiberi oportere, cujus exi-

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exiſtentia per clariſſima experimenta ad oculum demonſtrari non poſſit. Quæ lex, etſi primum ad compeſcendam libidinem pro arbitrio ac ſine cauſa urgente materias quasvis ſubtiles fingendi, merito eſt ſancita, tamen nunc quidem a plerisque ita extenditur, ut pæne omnis cauſarum cognitio funditus tollatur.

§. VII. Hoc loco autem iſti circumſpecti naturæ ſcrutatores primum declarent, an plane negent ullam in mundo exiſtere materiam tam ſubtilem, quæ neque ſenſus perceptibili modo afficiat, neque per experimenta eo reduci queat, ut quaſi manibus palpari poſſit. Quæ opinio, uti per graviſſimas rationes oſtendi poſſet abſurdiſſima, ita vix quenquam fore credo, qui eam aperte ac ſerio tueri velit. Ita igitur ſententiam ſuam mitigabunt, ut non licere dicant in explicatione cujusquam phænomeni naturæ, ad ullam materiam confugere, quæ ſenſibus percipi nequeat; hocque non quaſi ejusmodi materia ſubtilis non detur, ſed quod ejus exiſtentia ſenſibus probari non poſſit. Quod ſi ergo cujuspiam phænomeni cauſa in ejusmodi materia imperceptibili lateat, quod ſane fieri poſſe, niſi in prius abſurdum recidere velint, negare non poterunt, quomodo ſe Phyſicum gerere oportebit? Num cauſam ibi, ubi non eſt quærere, an forte omnino ne quærere quidem velle debebit; quorum illud homine, hoc vero Phyſico ita eſt indignum ut quod maxime. Quid igitur his cautis Philoſophis, quod reſpondeant, relinquatur non video, niſi vel ejusmodi phænomena ſine ulla cauſa fieri dicere, vel ad qualitates occultas revocare velint, quorum utrum Phyſico ſit turpius, tam facile non dixerim. Sin autem ſubſtantiæ cuidam immateriali et quaſi ſpiritui cauſam adſcribendam eſſe putent, ſibi ipſi minime conſtant, qui dum ideo omnem materiam ſub-

Exiſtentia materiæ ſubtilis inviſibilis aſſeritur.

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tilem ex Phyſica profligant, quod ſenſus non titillet: propter hoc ipſum ſpiritus introducendos eſſe arbitrantur, quod ſenſus noſtros prorſus effugiant. Ita fit ut, dum nihil, niſi cujus exiſtentia ſenſibus probari poſſit, in Phyſica admittere volunt, iidem ipſi innumerabilia ſpirituum genera introducant, quorum exiſtentia neque per experimenta nec per rationem unquam evinci queat: qua certe philoſophandi ratione, nihil, quod magis eſſet ridiculum, excogitari poteſt.

In ſtatuenda materia ſubtili phyſicum circumſpectum eſſe oportet.

§. VIII. His ſummis incommodis perpenſis, univerſa tandem iſta ſevera præceptio eo reducetur, ut in ſtatuendis ejusmodi materiis ſubtilibus, per quas phænomena naturæ explicentur, Phyſicum admodum cautum eſſe oporteat: quo ſenſu ſi accipiatur, equidem ita aſſentior, ut in hoc præcepto firmiſſimum totius Philoſophiæ præſidium poſitum eſſe exiſtimem. Non quod putem majorem hypotheſes atque materias ſubtiles fingendi libertatem cognitioni veritatis eſſe pernicioſam; mihi enim omnino perſuaſum eſt, nonniſi poſt plurima tentamina, quæ hypotheſibus excogitandis inſtituantur, ad veritatem nobis pertingere licere: ſed ſumma circumſpectio in hoc verſari debet, ut nullam ejusmodi materiam ſubtilem, qualem animo concepimus, in mundo actu exiſtere credamus, antequam certiſſime habeamus exploratum, ejusmodi materiam non ſolum eſſe poſſibilem, ſed etiam cunctis phænomenis mechanice explicandis eſſe aptam. Quamdiu autem nobis ad hunc certitudinis gradum pervenire non datur, ejusmodi hypotheſes ita tolerari conveniet, ut ad res tantum probabiles referantur.

Virtutem magneticam a materia

§. IX. Si jam hac adhibita cautela ad magnetem revertamur, ejus effectus ſtatim luculenter oſtendunt, ſubſiſtere

ſtere quoque extra magnetem materiam quandam ſubtilem, quæ phænomenorum cauſam in ſe complectatur. Neque vero, dum hoc aſſumo, hypotheſin mihi fingere videor: cum enim eæ res, quæ in ſenſus incurrunt, his phænomenis producendis non ſufficiant, neceſſe eſt, ut a materia quadam ſubtili ſenſus noſtros effugiente efficiantur: quod cum a nemine in dubium vocari poſſit, concedi omnino debebit, ejus modi materiam ſubtilem certiſſime actu exiſtere. Et quanquam plurimi, qui hanc de magnetis quæſtionem ſunt aggreſſi, voti non facti ſunt compotes, tamen nullum in hoc deceptum eſſe arbitror, quod materiam ſubtilem ſtatuerit: ſed potius, quod quisque ſuo arbitratu naturam atque indolem hujus ipſius materiæ conſtituerit; cum nihil aliud admittere debuiſſent, niſi quod ipſa phænomena hincque legitime illatæ concluſiones ineſſe manifeſto oſtendiſſent. Quam regulam ſi omnes naturæ ſcrutatores diligentius obſervaſſent, ſcientia certe naturalis non tantopere erroribus ac tenebris ſcateret, ut quid verum ſit quidve falſum, vix diſcernere liceat. quadam ſubtili oriri oſtenditur.

§. X. Quo autem clarius ſententiam meam de cauſa virtutis magneticæ exponam, primum aperiam, quemadmodum equidem cum ſtructuram magnetis ac ferri internam, tum indolem materiæ illius ſubtilis, cujus exiſtentia jam eſt demonſtrata, comparatam eſſe ſtatuam. Deinde vero ſimul rationes afferam, cur hoc potius modo utriusque ſtatum concipi debere credam quam ullo alio; ſicque ſententiam, quæ initio, me non repugnante, inſtar hypotheſis ac meræ fictionis ſpectari poteſt, ita confirmabo, ut non ſolum poſſibilis, ſed etiam veriſimilis videri incipiat. Tertio autem fuſius explanabo, quomodo ex his poſitis principiis omnia atque ſingula magnetis phænomena ſecundum leges naturæ Principia præſentis theoriæ breviter ob oculos ponuntur ac ratio inſtituti declaratur.

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turæ tam dilucide conſequantur, ut etiamſi adhuc eſſent incognita, tamen per ſolam theoriam a priori prædici ac definiri poſſent. Quod ſi præſtitero, hypotheſis, quæ modo probabilis erat facta, ſummum probabilitatis gradum, hoc eſt certitudinem plenam nanciſcetur, atque adeo hypotheſis eſſe ceſſabit, in numerum rerum actu exiſtentium tranſitura: hocque pacto nullum amplius ſupererit dubium, quin veram omnium magnetis proprietatum aſſignavero cauſam. Quocirca Judices ab Illuſtr. Academia Regia conſtitutos rogo, ut has meas meditationes debità cum attentione legere atque examinare velint.

Magnetem ratione pororum a reliquis corporibus potiſſimum diſcrepare.

§. XI. Quodnam ſit illud diſcrimen, quod magnetem a reliquis corporibus diſtinguit, atque aptum reddit ad tam inſignes & mirabiles proprietates recipiendas, quæ in reliqua corpora ſolo ferro excepto, nullo modo cadere queant; id quidem mihi per excluſionem certiſſime definiri poſſe videtur. Cum enim ad hæc phænomena producenda materia ſubtilis extra magnetem ſubſiſtens requiratur, hujus reſpectu in ipſo magnete non niſi pororum, per quos materiæ ſubtili tranſitus vel concedatur vel denegetur, ratio haberi poterit. Quod igitur magnes tam ſingulari præditus ſit proprietate, id a configuratione pororum ejus venit, in qua propterea poſitum eſt illud, quod quærimus, diſcrimen magnetem a reliquis corporibus diſtinguens. Quo diligentius autem omnes magnetis effectus perpendimus, eo magis in hac ſententia confirmamur: maxime autem hoc elucet, ſi attendamus cum ferro per ſolum contactum virtutem magneticam communicari poſſe; quo contactu cum neque addatur neque auferatur quicquam, neque etiam magnes de virtute ſua quicquam amittat, nihil aliud reliquum eſt, cui effectum tribuamus, præter pororum conformatio-

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mationem; in qua igitur præcipuum diſcrimen magnetis a reliquis corporibus eſt poſitum. Hanc eandem veritatem Celeb. Muſſchenbroeckius adeo agnoſcit, etſi conſtanter neget effectus magneticos mechanice explicari poſſe: qui cum omnibus hypotheſibus bellum perpetuum indixiſſet, ejus teſtimonium, ſi ullo ad ſententiam corroborandam egerem, plurimum ponderis habere debebit.

Solos poros non ſufficere ad phænomena producenda, ſed materiam ſubtilem præterea equiri.

§. XII. Hinc autem porro Sollertiſſimus iſte Obſervator recte concludit, a ſola pororum figura ea phænomena, quæ in magnete conſpicimus, minime oriri poſſe, quod quidem ita certum eſt, ut nulla probatione indigeat. Eatenus enim tantum a poris effectus ſenſibilis reſultare poteſt, quatenus materiæ cuipiam fluidæ per eos vel tranſire vel non tranſire liceat. Quamobrem præter certam pororum magnetis figuram neceſſario ſtatui debebit materia quædam fluida, quæ cum ſenſibus non percipiatur, jure merito ſubtilis vocatur, neque tamen ideo minus certe exiſtit, quam ſi ipſam manibus palpare liceret. In hoc enim negotio ſane rationis judicium plus valere debet, quam ſenſuum teſtimonium, quippe quod niſi ſimul ratione ſuffulciatur, nullam omnino vim retinet; cum contra ratio etiam ſine ſenſibus ad profundiſſimas veritates penetrare valeat. Hoc igitur loco a Muſſchenbroeckio diſſentire cogor, qui cum pororum formas phænomenis producendis impares agnoviſſet, neque tamen ad ullam materiam ſenſus non afficientem confugere auſus eſſet, ſibi viam atque aditum ad cognitionem cauſæ penitus præcluſit.

Pori magnetis materiæ ſubtili non quaqua verſus tranſitum concedunt.

§. XIII. Quemadmodum igitur pori reſpectu iſtius materiæ ſubtilis ſint comparati, ante omnia eſt definiendum. Ac primo quidem perſpicuum eſt, ſi pori magnetis penitus eſſent impervii materiæ ſubtili, tum præcipua magnetis phæno-

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phænomena, quæ in directione ſecundum certam plagam conſiſtunt, nullo modo produci poſſe: foret enim magnes utique ad omnes ſitus indifferens. Idem eveniret, ſi magnes quaqua verſus materiæ ſubtili transitum liberrime concederet; pariter enim nulla eſſet cauſa, cur magnes unum potius ſitum affectaret, quam ullum alium; cum in omni ſitu materia ſubtilis æque facile permeare poſſet. Cum itaque magnes neque prorſus impervius ſit huic materiæ ſubtili, neque liberrime tranſitum concedat, nil aliud reliquum eſt, niſi ut ſtatuamus, magnetem tranſitum quidem huic materiæ ſubtili concedere, at non liberrime, neque quaquaverſus æquali cum facilitate. Hoc modo jam conſequimur id, in quo cardo rei verſatur, quod magnes non ad omnes ſitus ſit indifferens, ſed perpetuo unum præ reliquis omnibus affectare debeat.

Materia ſubtilis ſecundum unicam directionem per magnetem fluere poteſt.

§. XIV. Pori igitur, per quos materia ſubtilis fluere poteſt, in magnete ſecundum certam quandam directionem erunt diſpoſiti, ita ut materia ſubtilis nonniſi ſecundum hanc directionem corpora magnetica permeare valeat. Talem ſtructuram clariſſime evincit conſtans illa, quam magnes affectare ſolet, directio: niſi enim in ipſo magnete pori ejusmodi conſtantem directionem ſequerentur, ratio ſane foret nulla, cur magnes unam præ reliquis poſitionem deſideraret. Hinc igitur pori in magnete ſecundum certam ac determinatam directionem meatus ſeu canales efformare videri poſſent, qui materiæ ſubtili ita tranſitum præbeant, ut ea ſecundum alias directiones permeare nequeat. Attamen hoc modo duæ remanerent directiones ſibi e diametro oppoſitæ, ad quas magnes induendas æque proclivis eſſe deberet. Concipiantur enim iſti meatus ſecundum directionem AB, efformati, atque perſpicuum erit, materiam ſubtilem æque ſecundum directionem AB atque contrariam BA per magnetem

Fig. 1

tem fluere poſſe. Quod cum experientiæ adverſetur, qua conſtat magnetem ſeu acum magneticam non in duplici ſitu quieſcere poſſe, neceſſe eſt, ut iſti meatus non ſint utrinque ſimiles, ſed ita diſſimiles, ut dum materiæ ſubtili tranſitum ſecundum alteram directionem AB concedunt, eidem tranſitum ſecundum contrariam directionem BA denegent.

Structura meatuum magneticorum ex analogia determinatur.

§. XV. Cum igitur meatus magnetici materiæ ſubtili tranſitum quidem permittant, reditum autem in eadem directione recuſent, ſimiles propemodum erunt canalibus in corpore animali, qui in ſua cavitate ſanguinem aliaque fluida devehunt, regreſſum vero non admittunt, qui effectus valvularum ope obtinetur. Quoniam ergo natura in ſuis operationibus conſtantes obſervat leges, atque ad ſimiles effectus producendos ſimilibus perpetuo utitur cauſis, dubitare non licet, quin reverſio materiæ ſubtilis in meatibus magneticis per ſimilem machinationem impediatur. Hanc ob cauſam leges naturæ atque adeo veritatem ſtudioſiſſime mihi quidem ſequi videor, ſi meatus magneticos ſimilibus valvulis inſtructos ſtatuam, quibus efficiatur, ut materia ſubtilis in A ſatis libere intrare et ad B exire queat, contra vero ipſi ad B ingreſſio et progreſſio verſus A per has valvula præcludatur. Qua ratione autem hæ valvulæ ſint conſtructæ, noſſe non admodum refert; dummodo regreſſui materiæ ſubtilis coercendo ſint aptæ. Interim tamen veriſimillimum eſt, eas a villis ſeu fibrillis tenuiſſimis internam meatuum cavitatem obſidentibus atque ab A verſus B reclinatis efformari. Hujusmodi ſtructura non ſolum ob ſimplicitatem veritati maxime eſt conſentanea, ſed etiam per ea experimenta, quibus virtus magnetica tam facile produci, iterumque deſtrui poſſe oſtenditur pleniſſime confirmatur.

Fig. 2.

Pororum conformatio in magnete ac ferro ſimulque in reliquis corporibus exponitur.

§. XVI. Præcipuum igitur diſcrimen, quo magnes ac ferrum a reliquis corporibus diſtinguitur, ita in pororum diſpoſitione eſt poſitum, ut magnetis et ferri magnetica virtute jam imbuti pori conſtituant meatus utrinque quidem apertos, at valvulis ſeu villis ita inſtructos, ut materia ſubtilis ad alteram tantum extremitatem intrare, et per totum meatum progreſſa in altera extremitate exire queat. In ferro autem virtutis magneticæ experte inſunt quidem pori illi fibrillis valvularum vices ſuſtinentibus obſiti, verum nondum ita ordinati ut meatus continuos, in quibus fibrillæ illæ ubique ſecundum eandem plagam ſint reclinatæ, conſtituant. Hac corporum magneticorum ſtructura evicta, manifeſtum eſt reliqua corpora omnia ejusmodi poris ac meatibus carere debere; quamobrem vel quaquaverſus materiam illam ſubtilem liberrime tranſire ſinunt, vel omnino ipſi erunt impervia. Experientia autem manifeſto docet, materiam iſtam ſubtilem, a qua virtus magnetica pendet, reliqua corpora omnia liberrime permeare; propterea quod effectus magnetis a corporibus quibuscunque interpoſitis prorſus non impeditur: quod fieri non poſſet, niſi materia ſubtilis liberrimum tranſitum per omnia corpora non magnetica inveniret.

Materia ſubtilis meatus magneticos permeans eſt pars aetheris

§. XVII. Quod porro ad materiam illam ſubtilem attinet, quæ meatus magneticos percurrendo virtutem magneticam dictam producat, eam primum ab aere eſſe diverſam dubitari omnino nequit; cum quod ea reliqua corpora liberrime permeat, tum vero maxime, quod eadem magnetis phænomena in vacuo obſerventur. In aethere igitur iſta materia ſubtilis reſideat neceſſe eſt, ſi quidem univerſum fluidum maxime elaſticum undique diffuſum aetheris nomine complectamur. Non ſolum autem veriſimile

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mile eſt, univerſum aetherem non ex materia homogenea eſſe conſlatum; ſed mox ex ipſis magnetis phænomenis fuſius oſtendetur, duplicem ſaltem materiam, alteram craſſiorem, alteram ſubtiliorem in æthere ſtatui debere, quarum hæc tantum, quæ eſt ſubtilior, per meatus magneticos tranſire valeat, altera craſſiore penitus excluſa. Interim hoc diſcrimen non impediet, quo minus utraque materia aetheris aequali vi elaſtica ſit prædita, ita ut univerſus aether hoc non obſtante ubique eadem atque aequabilili elaſticitate gaudeat. Cum igitur hæc hypotheſis nullam contradictionem involvat, ob rationes mox allegandas ſtatuo præcipuam virtutis magneticæ cauſam in aetheris parte ſubtiliſſima eſſe poſitam, atque meatus magneticas tam eſſe arctos, ut iſtam partem ſubtiliſſimam tantum transmittant, craſſioribus autem moleculis ſint impervii. Reliqua autem corpora omnia aetheri tranſitum liberrime concedere extra dubium eſt poſitum.

§. XVIII. Eſt itaque aether fluidum heterogeneum ex particulis diverſæ molis conſlatum, ſimili modo, quo aerem ex diverſis materiis compoſitum eſſe conſtat; nihilo vero minus aether perinde ac aer, ſi variæ iſtæ particulæ aequabiliter fuerint inter ſe permiſtæ, fluidum homogeneum mentietur: ſin autem iſtæ materiæ a ſe invicem fuerint ſecretæ, tum utique fluida conſtituent heterogenea, quæ etiamſi ratione elaſticitatis in æquilibrio conſiſtant, tamen non niſi difficulter rurſus inter ſe permiſceantur. Similis ſcilicet conditio in omnibus fluidis, quæ ex moleculis diverſæ magnitudinis conſtant, deprehenditur; quemadmodum in aqua et oleo, vel etiam in aere et aqua manifeſtum eſt, quæ materiæ etſi in debita ratione permiſtæ conſiſtere poſſunt, tamen a ſe invicem ſegregatæ difficulter ſe iterum permiſceri patiuntur. Hic itaque nihil aſſumo, niſi quod

Materias aetheris diverſas difficulter uniri.

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cum veritati maxime ſit conſentaneum, tum vero etiam conſtanter a natura effici obſervetur. Quamobrem hanc mihi hypotheſin concedi jure equidem poſtulare poſſem, etiamſi nullas præterea rationes afferrem: cum autem clariſſime ſim oſtenſurus, omnia magnetis phænomena tam facile ac legibus naturæ convenienter hinc explicari poſſe, hanc indolem aetheris etiam demonſtraſſe jure mihi videbor.

Atheris pars tantum ſubtilior per meatus magneticos transfluit.

§. XIX. Hæc jam ſunt duo illa principia, in quibus conjunctis cauſa omnium effectuum magneticorum continetur, quorum alterum in peculiari magnetis ac ferri ſtructura eſt poſitum, alterum vero in aetheris natura. In magnete nimirum plurimi inſunt meatus filamentis quaſi valvulis obſiti, per quos ſubtilisſimæ aetheris particulæ tranſire queant, craſſiores vero penitus excludantur. Quoniam vero non totus magnes hujusmodi habet ſtructuram, ut ipſi nihil cum reliquis corporibus ſit commune, præter hos meatus magneticos aliis quoque poris amplioribus ſcatebit, qui non ſolum materiæ aetheris ſubtiliori ſed etiam craſſiori liberum tranſitum concedant. Ætheris porro indoles facilius percipietur, atque adeo mechanice explicari poterit, ſi cum ex infinitis vorticulis, in quibus materia ſubtiliſſima rapidisſime in gyrum agatur, compoſitum concipiamus, cujusmodi ſtructuram ſumma aetheris elaſticitas declarat: ſpatia autem anguloſa, quæ inter iſtos vorticulos relinquuntur, ab aliis vorticulis longe minoribus occupari veriſimillimum eſt; atque hi vorticuli minores materiam illam aetheris ſubtiliorem meatus magneticos percurrentem exhibere mihi quidem videntur. Hinc autem difficultas, qua iſti minores vorticuli, ſi a reliquo aethere ſemel ſint ſeparati, ſe iterum inſinuent, perſpicue intelligitur,

§. XX.

§. XX. Hiſce duobus ſtabilitis principiis videamus, quid ex iis conjunctis conſequi debeat. Ac primo quidem manifeſtum eſt, præter magnetem omnia corpora in aethere nullum ſenſibilem effectum eſſe productura, cum neque quieti neque motui ejus obſiſtant: ita ut aether in eodem fere ſtatu ſit permanſurus, ſive inſint in ipſo corpora non magnetica ſive minus. At vero corporum magneticorum aethere circumdatorum longe alia erit ratio propter illos meatus, qui in altero tantum termino parti aetheris ſubtiliori introitum permittunt; ob quam ſingularem ſtructuram fit, ut aether hæc corpora ambiens in aequilibrio eſſe nequeat. Propter ſummam enim elaſticitatem aether in hiatus iſtorum meatuum fortiſſime premet, et quoniam a parte oppoſita hæc preſſio compeſci nequit, neceſſe eſt, ut particulæ aetheris ſubtiliores in meatus illos magneticos, qua introitus patet, ingenti vi irrumpant; quo pacto aetheris partes ſubtiliores a craſſioribus ſecernentur. Tum vero hæ particulæ penetratis meatibus in alteris terminis prorumpent, ubi quia ſeſe cum aethere ambiente ſubito permiſceri non patiuntur, quaſi a fluido heterogeneo reflectentur, motumque, qua minime reſiſtitur, tamdiu conſervabunt, quoad ab aethere ambiente ſenſim abſorbeantur, ſinibusque angularibus iterum includantur.

Effectus materiæ ſubtilis in magnetem generatim exponitur.

§. XXI. Quo hæc, quippe in quibus omnis cardo præſentis quæſtionis verſatur, diligentius evolvantur, ordiamur ab ipſa terra tanquam fonte et communi receptaculo tam magnetis quam ferri. Abundabit ergo terra propter ingentem mineræ tam magnetis quam ferri copiam in viſceribus incluſam, maxime talibus meatibus, quales in unoquoque magnete ineſſe oſtendi. Et hancobrem cum primum terra initio aethere fuerit circumfuſa, neceſſe eſt, ut materia

Vortex magneticus circa terram efformari debuit.

teria illius ſubtilior hos terræ meatus magneticos ſtatim ſit ingreſſa: id quod maxima cum celeritate eſt factum, quia in has cavitates quaſi in loca vacua ſumma vi irrupit. Tanta ergo celeritate emenſis ſingulis meatibus materia hæc ſubtilis in ætherem ambientem proſiluit, ab eoque tanquam ab obice firmo eſt reflexa, ſimili fere modo quo aer contra aquam fortiſſime impulſus repercutitur, atque ad latera deflectitur. Quare hæc materia qua minimam invenit reſiſtentiam, moveri perrexerit neceſſe eſt; ad latera autem defluere debuit, quoniam reverti, unde venerat, tam ob ſtructuram meatûum, quam ob materiam ſimili vi inſequentem, minime potuit. Facile igitur ad latera defluens ad orificia meatuum revertetur, in quæ initio intraverat; ubi non ſolum nullam reſiſtentiam offendet, ſed quia liberrime ingredi poteſt, eo undiquaque preſſa quaſi attrahetur. Cum igitur fuerit denuo in hos meatus ingreſſa atque ſimili modo reverti debeat, mox flumen continuum ſeu vorticem perrennem circa terram formare debuit; propterea quod materia ſubtilis ex meatibus erumpens veſtigia antecedentis continuo ſequendo quaſi ſponte ad orificia meatuum perducitur, hocque motu periodico ſine ulla intermiſſione agitatur.

Fig. 3. Formatio vorticis circa terram exiſtentis uberius explicatur.

§. XXII. Si igitur globus A B repræſentet tellurem, in qua meatus magnetici ab A ad B ſint diſpoſiti, ita ut materia ætheris ſubtilior ad A ingrediatur, in B vero exeat; tum ob rationes expoſitas materia ſubtilis ad B erumpens utrinque deflectet, atque ad C & C circumfluendo revertetur ad A, ubi denuo in meatus magneticos intrabit, ſicque vorticem permanentem circa terram conſtituet. Neque tamen perpetuo eadem materia ætheris ſubtilior ad hunc vorticem formandum impendetur, ſed in C & D continuo quædam particulæ cum æthere circumfuſo ſeſe permiſcebunt; quæ jactura

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jactura autem ſtatim per novam ſimilis materiæ ſubtilioris ab æthere ſecretionem ad A factam reſarcietur: ita ut non obſtante perpetua permixtione, quæ fit dum materia ſubtilis extra meatus per ætherem revolvitur, vortex tamen perennis conſervetur. Erunt igitur A & B ambo terræ poli magnetici, qui cum a diſpoſitione meatuum magneticorum AB pendeant, neque cum veris terræ polis ſingularem habeant connexionem, mirum non eſt, quod poli magnetici a polis mundi diſcrepent: unde declinationis magneticæ cauſa potiſſimum originem trahit; quæ autem in ſequentibus accuratius perpendetur.

§. XXIII. Quoniam ſi terra tolleretur, æther in regione ABCD fere quieſceret, quæſtio ſponte ſe offert, unde, terra in AB conſtituta, tam vehemens motus in materia ætheris ſubtilior oriatur: conſtat enim motum ſine diſpendio virium omnino produci non poſſe. Cum igitur demonſtraverim iſtum motum a vi ætheris elaſtica generari, neceſſe eſt ut hæc vis elaſtica circa terram ſenſibiliter diminuatur; atque ſatis probabile videtur, hanc diminutionem vis elaſticæ reciproce proportionalem eſſe diſtantiis a centro terræ. Hoc autem conceſſo cauſa gravitatis tanto ſtudio anquiſita ſit maxime obvia: ſit enim corpus Pp ad diſtantiam OP a centro terræ O poſitum; cujus craſſities Pp reſpectu diſtantiæ OP quaſi ſit nulla. Quod ſi jam vis ætheris elaſtica abſoluta ponatur $= E$, erit hæc vis diminuta in

Cauſa gravitatis indicatur.

Fig. 4.

$P = E - \frac{A}{OP}$, & in $p = E - \frac{A}{Op}$. At illa vi corpus deorſum, hac vero ſurſum urgetur; unde vis, prævalens, quæ corpus deorſum urgebit, erit $= \frac{A}{Op} - \frac{A}{OP} = A.$

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$= \frac{A.\,Pp}{Op.\,OP}$, ideoque proportionalis reciproce quadrato diſtantiæ corporis a centro O. Hic autem locus non eſt hæc fuſius perſequendi, ac pro rei dignitate confirmandi: interim tamen hæc tranſitio plurimum valere debebit ad theoriæ præſentis veritatem evincendam; propterea quod ſolius veritatis hoc eſt proprium, ut cum omnibus phænomenis perfectiſſime conveniat.

Gravitas univerſalis corporum mundanorum explicatur.

§. XXIV. Cum igitur gravitas inde oriatur, quod in vicinia terræ vis ætheris elaſtica debilitetur ob vorticem magneticum circa terram formatum; perquam veriſimile eſt ſolem atque planetas pariter ejusmodi meatibus magneticis abundare, hincque circa corpus cujusque ſimilem vorticem materiæ ætheris ſubtilioris exiſtere, quo vis elaſtica ætheris in vicinia horum corporum pro inverſa diſtantiarum ratione diminuatur, ſicque gravitas univerſalis rationem diſtantiarum inverſam duplicatam ſequens efficiatur. Hæc certe gravitatis explicatio, ſicuti quaſi ſponte ſe obtulit, ita ſtatim omnibus reliquis explicationibus, quæ adhuc ſunt excogitatæ, facile palmam præripit, quia non ſolum non tantis difficultatibus quam reliquæ premitur, ſed etiam tam facile atque naturæ convenienter omnia phænomena feliciſſime explanat. Non dubitarem quoque ex hoc eodem principio cauſam virtutis electricæ dilucide explicare: verum quoad hæc mihi fuſius evolvere liceat, in eum tantum finem hæc annotare viſum eſt, quo veritas ſyſtematis, in quo corroborando ſum occupatus, eo magis eluceret, atque adverſus omnes objectiones, quas equidem non admodum pertimuerim, firmius conſiſtat.

Vorticis magnetici motus curatius definitur.

§. XXV. Quod igitur ad vorticem illum materiæ ſubtilis circa terram formatum attinet, ſtatim manifeſtum eſt

eſt celeritatem materiæ ſubtilis in meatibus AB, ubi a reliquo æthere omnino eſt ſeparata, eſſe maximam, ideo quod a ſumma vi elaſtica ſollicitata in his meatibus fere nullam ſentit reſiſtentiam. Quando vero ad B ex his meatibus prorumpit, ab æthere circumfuſo quaſi reflectitur, & directionem ſuam ad latera C & D inflectere cogitur, tum ejus celeritatem vehementer diminui oportet. Hanc ob cauſam flumen hujus materiæ ſubtilis extra terram plurimum amplificabitur, perinde ac fluvius ubi minori celeritate progreditur, in majus ſpatium ſe ſimul expandit, ita ut in quovis loco celeritas ſit amplitudini ſpatii, quod occupat, reciproce proportionalis. Huc autem accedit, quod cum hac materia ſubtiliori extra meatus magneticos mota non exigua ætheris naturalis portio permiſceatur; quo fit, ut volumen vorticis extra terram eo magis augeri, contra vero celeritas diminui debeat. Fig. 5.

§. XXVI. Rationes igitur hæ ſatis dare evincunt motum materiæ ſubtilis cum reliquo aethere jam permixtæ admodum lentum eſſe debere, quocirca ab hoc vortice quamvis motus ſit curvilineus, tamen nulla ſenſibilis vis centrifuga, qua effectus gravitatis turbari queat, ſe manifeſtare poterit. Hoc igitur maxime mea explicatio tam virtutis magneticæ quam gravitatis atque adeo attractionis univerſalis a reliquis ſe diſtinguit, quod dum alii omnes vires in vi centrifuga poſitas ſtatuant, ego huic vi nullas vices tribuam; ſed hæc omnia phænomena a ſola vi elaſtica aetheris derivem: unde ſimul corporum cohæſionem ac duritiem pendere, omnino dubitari nequit, quo ipſo theoriæ meæ maximum firmamentum accedere nemo inficiabitur. Appellabo autem in poſterum materiam illam aetheris ſubtiliorem meatus magneticos pervagantem, diſtinctionis cauſa materiam

Status aetheris terram ambientis exponitur.

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riam magneticam, quæ cum in aethere naturali ſatis raro ſit diſperſa, tamen in vicinia terræ in majori portione cum aethere erit permixta, eo quod continuo per meatus magneticos a reliqua aetheris maſſa ſecernitur. Hanc obrem univerſa terra circumfuſa erit fluxu perpetuo aetheris multo majori copia materiæ magneticæ imprægnati, quam quidem in aliis regionibus a terra aliisque corporibus mundanis multum remotis in ſe complecti ſolet.

Origo meatuum magneticorum traditur.

§. XXVII. Si jam originem meatuum magneticorum in terra formatorum ſpectemus, perinde erit ſive dicam eos ſimul cum terra eſſe creatos, ſive ſucceſſu temporis demum ab ipſa materia magnetica effectos: verum tamen poſterius veritati magis videtur conſentaneum. Cum enim iſti meatus conſtent poris magneticis ſecundum certam directionem diſpoſitis atque valvulis inſtructis, etiamſi ſi pori initio nondum ita fuiſſent ordinati, tamen quia ſunt mobiles, a materia magnetica, cum ſimel fuerit ingreſſa, facile ſecundum certam directionem diſponi, ſicque in meatus continuos efformari potuerunt. Magna namque debet eſſe vis materiæ magneticæ, cum ſemel poros corporum magneticorum intrare coeperit, ob ſummam celeritatem, qua tum movetur. Hacque vi, ſi percurſo quopiam ſpatio poros minus congrue diſpoſitos offenderit, eos facile ad ordinem dirigere, atque filamenta tenuiſſima, quibus pori ſunt obſiti, ita inflectere valuit, ut meatus continuos conſtituerent. Statuendum igitur eſt, id quod veriſimillimum aliunde conſtat, per totam terram corpora magnetica continuo tractu et extitiſſe et etiamnunc exiſtere; neque tamen hinc ſequitur meatus magneticos ubique ſecundum lineas rectas progredi, ſed ſufficit dummodo ab uno termino ad alterum ſint continui. Multo minus ex his colligi poteſt, polos

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polos magneticos terræ ſibi ediametro eſſe oppoſitos, quin potius fieri poteſt, ut in terra plures duobus polis magneticis exiſtant, de quo infra videbo. Interea hinc ſatis tuto concludere licet, quoniam meatus magnetici per terræ viſcera continuo tractu extenduntur, terram intus non eſſe excavatam, uti nonnullis philoſophis eſt viſum.

§. XXVIII. Meatus iſti magnetici in terra ſemel efformati perpetuo in eodem ſtatu permanere deberent, ſi quidem terra quieſceret, neque in viſceribus ejus ulla ſive deſtructio ſive generatio corporum magneticorum eveniret. Cum autem terra ingentibus mutationibus etiam in imis viſceribus ſit obnoxia, meatus magneticos hinc quoque aliquam alterationem perpeti debere, maxime probabile eſt. Imprimus vero motus terræ diurnus non exiguam mutationem in meatibus magneticis producere debebit. Quia enim terra intervallo unius diei circa axem ſuum ab occaſu in ortum rotatur, effectus aetheris idem erit, ac ſi terra quieſceret, aether autem aequali celeritate ab ortu in occaſum circumferretur. Ob hunc motum aether in meatus magneticos oblique incurrens vim exercebit ad meatuum directionem immutandam, hincque polos terræ magneticos ab ortu ad occaſum promovendos. Quoniam vero pori magnetici facile ſe mutari patiuntur, uti ex ferri phænomenis concludere licet, ejusmodi polorum magneticorum variatio ſucceſſu temporis actu animadverti debebit, eritque iſte effectus eo ſenſibilior, quo longius poli magnetici a polis mundi fuerint remoti. Ex his itaque cauſa vera mutabilitatis polorum magneticorum terræ, hincque oriundæ mutabilitatis declinationis magneticæ clariſſime explicatur.

Meatus magnetici in terra ſunt mutabiles.

C 3 §. XXIX.

In terra plures duobus polis magneticis ineſſe poſſunt.

§. XXIX. Quanquam hactenus duorum tantum polorum magneticorum terræ mentionem feci, tamen ex iisdem principiis facile intelligetur, ſtructuram terræ internam ita comparatam eſſe poſſe, ut plures polos exhibeat, atque adeo Halleji ſyſtema cum hac theoria egregie conſiſtere poſſit, qui prope utrumque polum mundi binos polos magneticos ſtatuit. Si enim terra circa centrum et axem per notabile intervallum magneticis corporibus careat, ita ut meatus magnetici A B et ab ſenſibiliter ſint a ſe invicem ſejuncti, tum poli A et a, itemque ex altera parte B et b confundi atque unum polum conſtituere non poterunt: ſed quatuor polos diverſos repræſentabunt, qui a polis mundi P et p vel aequaliter vel inæqualiter diſtabunt, prout directiones meatuum fuerint axi Pp parallelæ vel ſecus: atque ipſa ſyſtemata meatuum A B et ab magis minusve ab axe ſint remota. Cum autem hæc ab interna conſtitutione terræ pendeant, a priori minime determinari poterunt, ſed ex ipſis phænomenis concludi debent; quod negotium obſervationes circa declinationem et inclinationem acus magneticæ inſtitutæ, potiſſimum conficient. Sufficiat igitur poſſibilitatem plurium polorum magneticorum in terra oſtendiſſe, atque ex theoria docuiſſe hos polos propter motum vertiginis terræ continuo ab ortu in occaſum circumferri debere, quod ipſum cum experientia mirabiliter concordat.

Fig. 5.

Virtus ſingulorum magnetum a vortice terræ potiſſimum oritur.

§. XXX. Hæc in genere de univerſa tellure, quatenus virtute magnetica pollet, annotare viſum eſt, antequam theoriam in ſpecie ad magnetem ac ferrum applicarem; quoniam virtus magnetica, quæ in magnete ac ferro deprehenditur, maximam partem vortici illi materiæ magneticæ circa terram formato originem ſuam debet. Etſi enim in unoquoque

quoque magnete, ſi a terra remotus in æthere verſaretur, ſimilis materiæ ſubtilis motus periodicus et quaſi vortex naſci deberet; tamen is cum ob corporis parvitatem tum propter meatuum paucitatem reſpectu vorticis terreſtris maxime debilis atque adeo vix foret ſenſibilis: ſin autem magnes jam in vortice terræ ſit conſtitutus, multo majorem acquiret virtutem; propterea quod materia magnetica hoc loco valde abundat, neque demum a reliquo aethere ſecerni debet. Tum vero, in quo maximum poſitum eſt momentum, materia magnetica in vortice terræ jam motu eſt prædita, quo fit ut non ſolum in meatus magneticos majori vi irrumpat, ſed etiam hos meatus ſecundum ſuam motus directionem diſponere valeat; unde in magnete vis ſeſe verſus certas plagas convertendi naſcitur, quæ prorſus abeſſet, ſi magnes in aethere adhuc quieſcente verſaretur, quippe quo caſu magnes ad omnes ſitus foret indifferens. Hunc igitur duplicem vorticis terræ effectum in ſingulos magnetes diligentius perpendere conveniet, ut intelligatur, cujusmodi phænomena in unoquoque magnete ſolitario evenire debeant: quo facto, quemadmodum plures magnetes tum inter ſe tum ratione ferri affecti eſſe debeant inveſtigabo.

§. XXXI. Cum igitur materia ſubtilis magnetica in aethere terræ circumfuſo ingenti copia abundet, in cujusque magnetis meatus magna vi irrumpet, atque ob ſummam aetheris elaſticitatem vehementi rapiditate per eos fluet. Tanta igitur celeritate ex his meatibus quoque prorumpet, quam cum ob reſiſtentiam aetheris conſervare nequeat, primum ipſa celeritas ſtatim diminuetur, tum vero directio ad latera inflectetur atque ſtatum permanentum quærendo tandem ita motus temperabitur, ut ad orificia meatuum

Circa ſingulos magnetes vortex magneticus generatur.

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um revertatur. Continuo ergo tranſitu per meatus atque reditu vortex omnino ſimilis ei, quem circa terram generatum eſſe oſtendi, circa ſingulos magnetes efformabitur. In hujusmodi autem vortice minore unum magnetem ambiente materia ſubtilis magnetica multo inerit copioſior, quam in vortice terreſtri, eo quod non ſolum eadem materiæ magneticæ copia adeſt, ſed etiam per meatus magnetis continuo nova ſecretio iſtius materiæ ſubtilioris aetheris a craſſiore accedit. Quare cum virtus magnetica a copia materiæ magneticæ in quovis vortice contentæ pendeat, circa magnetes hæc virtus multo magis vigebit, quam circa terram, atque ob hanc rationem virtus magnetica terræ generalis etiam a minimo magnete facile ſuperatur, quod quamvis per experientiam ſit notiſſimum, tamen clarius perſpicietur, ſi in effectus virtutis magnetis, qui in mutua attractione et directione conſiſtunt, data opera inquiremus.

Magnes non poteſt eſſe ad omnes ſitus adiſſerens.

§. XXXII. Si materia ſubtilis magnetica circa terram quieſceret, tum ob ſummam elaſticitatem irrumperet quidem in cujusque magnetis meatus vorticemque formaret, verum nullam prorſus vim exereret ad magnetem ſecundum certam plagam dirigendum, cum magnes utcunque poſitus eodem modo reſpectu materiæ ſubtilis ſit affectus, neque vis aetheris elaſtica quicquam lucraretur, ſi magnetem in hanc potius quam aliam plagam dirigeret. Eſt autem hæc lucri commemoratio non vox inanis, verum in univerſa rerum natura maximum habet pondus: ubique enim obſervamus vires in mundo exiſtentes nil niſi lucri cauſa facere, atque effectum ſemper ita eſſe comparatum, ut eo conatus et quaſi appetitus virium ſollicitantium maxime expleatur; ſic funis ſuſpenſus, ſeu catena eam induit curvaturam, ut ejus

ejus centrum gravitatis infimum occupet locum, quoniam hoc modo conatus gravitatis maxime expletur. Atque ex hoc principio omnes quæſtiones naturales, etiamſi vias eas a priori reſolvendi nulla pateat, tamen per methodum maximorum ac minimorum feliciſſime reſolvi poſſunt; id quod pluribus exemplis jam luculenter a Geometris eſt oſtenſum, atque multis aliis novis oſtendi poſſet. Quamobrem ſi in determinatione effectus a materia magnetica in meatus incurrente oriundi aqua mihi ob defectum principiorum ſubinde hæreat, iſte defectus per illud principium univerſale convenientiſſime ſupplebitur, neque ullum erit periculum ob ignorantiam principiorum genuinorum in errores prolabendi. Quin etiam hoc modo plures alias tædioſi calculi evitari poſſunt.

§. XXXIII. Ex hac naturæ lege univerſali dijudicare licet, quomodo magnes in vortice terreſtri magnetico poſitus affici debeat. Ac primo quidem ſi a motu hujus materiæ ſubtilis abſtrahamus, ea, uti jam eſt oſtenſum, in meatus magnetis penetrabit, et verticem circa magnetem efformabit, qui multo magis materia ſubtili magnetica abundabit quam vortex terreſtris, in quo continetur. Quod autem ad motum vorticis terreſtris attinet, perſpicuum eſt ſi directio meatuum magnetis ita congruat cum motu materiæ ſubtilis, ut etiam curſum non inflectendo in meatus ingredi poſſit, tum motum fore celerrimum, et penetrationem meatuum minima difficultate perfici; quoniam et vis elaſtica et motus in materia ſubtili jam inſitus ad eundem effectum producendum concurrunt. Cum igitur in hoc ſtatu maximus effectus producatur, dubium eſt nullum, quin vortex terræ magneticus vi ſit præditus magnetes quosvis

Directio magnetis convenire debet cum motu materiæ magneticæ.

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in hanc ipſam directionem, in qua effectus oriatur maximus, dirigendi. Atque hoc æque certe evenire debere ſtatuendum eſt, ac ſi ipſæ vires, quibus iſta directio efficiatur, perfectiſſime eſſent cognitæ. Quamvis autem in hac ratione acquieſcere poſſemus, tamen quia in hac magnetum converſione omnis theoriæ cardo verſatur, operam dabo, ut clarius intelligatur, quomodo iſte effectus ſecundum leges mechanicæ producatur; quo magis ex conſenſu cauſæ finalis cum cauſa efficiente veritas theoriæ perſpiciatur atque confirmetur.

Converſio magnetis verſus certam plagam mechanice explicatur.

§. XXXIV. Conſideremus unicum meatum magneticum in vortice terreſtri libere poſitum: quod enim in uno meatu evenire debere oſtendetur, idem in pluribus atque ideo in integro magnete uſu venire debet. Teneat igitur meatus magneticus ſitum AB, atque materia ſubtilis vorticoſa hoc loco moveatur ſecundum directionem AC, cujus celeritas per hanc rectam AC exprimatur, ita ut directio meatus AB cum directione motus materiæ ſubtilis angulum conſtituat BAC; qui angulus ſi foret nullus, dubium non eſt, quin meatus in hoc ſtatu permanſurus eſſet. Conſideretur nunc celeritas, qua materia ſubtilis magnetica in meatum eſſet irruptura, ſi quiſceret, et a ſola elaſticitate urgeretur, et repræſentetur hæc celeritas per rectam AB. Quoniam igitur materia ſubtilis et proprium habet motum AC et ad motum AB ſuſcipiendum ſollicitatur, duplici hoc motu efficietur, ut materia ſubtilis annitatur promoveri ſecundum directionem diagonalis BD, completo parallelogrammo ABDC, cum celeritate per ipſam diagonalem AD expreſſa. Ex hoc conatu ſecundum directionem AD movendi naſcetur neceſſario vis meatum verſus

Fig. 6.

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hanc ipſam directionem convertendi, quæ vis cum a celeritate ipſa AD, tum ab angulo BAD pendebit. Cum igitur meatus AB in A urgeatur in directione AD, reſoluta hac vi in normalem ad AB & incidentem, illa, quæ erit ut AD. ſin BAD exhibebit vim convertentem. Quare cum ſit ex Trigonometria AD : AC = ſin BAC : ſin BAD, erit AD. ſin BAD = AC. ſin BAC: & hanc obrem vis meatum AB verſus directionem vorticis AC convertens erit ut AC. ſin BAC.

§. XXXV. Hinc ergo erit vis, qua meatus magneticus AB verſus directionem vorticis AC inflectitur in ratione compoſita ex celeritate materiæ vorticoſæ AC, & ſinu anguli BAC, quo ſitus magnetis a directione AC diſſidet. Cum igitur celeritas materiæ vorticoſæ AC maneat quaſi eadem, meatus magneticus quieſcere non poterit, niſi cum fuerit ſinus anguli BAC = o, quod quidem duobus caſibus fit altero quo angulus BAC = o; altero quo BAC = duobus rectis. Quamvis autem hoc poſteriori caſu quieſcere poſſit, tamen hic non dabitur ſtatus quietis permanens, ſed ſtatim ac meatus vel tantillum ab hoc ſitu declinatur, tum ſeſe in alterum æquilibrii ſtatum permanentem, ubi BAC = o, recipiet, ubi acquieſcere poterit. Maxime autem in hunc ſitum contendet, ſi ab eo ad angulum rectum diſtet, hoc eſt ſi angulus BAC fuerit rectus, tum enim hujus anguli ſinus fit maximus quippe ſinui toti æqualis; ſin autem angulus BAC fuerit ſive acutus ſive obtuſus vis erit minor. Præterea autem notandum eſt, quoniam non omnis materia ætheris circumfuſi per meatum tranſire poteſt, ſed tantum ejus pars ſubtilior, quam materiam magneticam voco, etiam copiam hujus materiæ in ſuperiorem expreſſionem AB. ſin BAC eſſe introducendam: ita ut ſit vis convertens proportionalis producto ex copia materiæ magneticæ circa magne-

Vis magnetem convertens determinatur.

tem exiſtentis, ejus celeritate, & ſinu anguli, quo directio meatus a directione motus vorticis diſtat.

Magnetum anomalorum ratio explicatur.

§. XXXVI. Quilibet igitur magnes, qui habet meatus ſuos inter ſe parallelos & fere in directum diſpoſitos, eum in vortice ſitum affectabit, ad quem unumquemque ſimplicem meatum urgeri oſtenſum eſt. Scilicet talis magnes, niſi ab aliena vi retineatur vel a gravitate impediatur, in eum ſitum ſe componet, in quo directio meatuum cum directione motus vorticis ita conſentiat, ut materia ſubtilis ſine ulla motus ſui inflexione in meatus intrare poſſit. Hinc autem excludendi ſunt magnetes compoſiti & quaſi anomali, vel ex pluribus magnetibus ſimplicibus conſtantes, vel in quibus meatus nec inter ſe ſunt paralleli, nec in directum formati. Hujusmodi magnetes idcirco plerumque plures duobus habent polos, ſimili modo quo terram quatuor polis præditam eſſe obſervationes declarare videntur. Quin etiam numerus polorum in eodem magnete impar eſſe poteſt, quod evenit ſi meatus, qui in altera extremitate disjuncti plures polos conſtituunt, iidem in altero termino uniuntur unicumque polum efformant. Sic magnes AB b, in quo meatus ad A uniti verſus B & b dirimuntur, tres habebit polos in A, & B ac b, quorum quisque eo erit fortior, quo plures meatus ad eum formandum concurrunt. Atque hinc etiam fieri poſſe intelligitur, ut unus tantum polus diſtinctus in magnete deprehendatur, quod eveniet, ſi meatus ad A concurrentes verſus alteram extremitatem ita divergant, ut nusquam tot coëant, quot ad polum diſtinctum repræſentandum requirantur. Cujusmodi igitur ſitum tales magnetes anomali in vortice terræ affectare debeant, facile colligere licet; cum enim ſinguli meatus ſitum

Fig. 7.

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tum intentum recipere nequeant, omnes conjunctim ejusmodi ſitum medium eligent, in quo vires dirigentes ſe mutuo in æquilibrio teneant.

§. XXXVII. Cum ergo magnetes anomali nullum amplius negotium faceſſant, ſi magnetum ſimplicium, in quibus meatus curſu parallelo ab uno termino ad alterum in directum extenduntur, duosque polos diſtinctos exhibent, ratio fuerit expoſita; ad hujusmodi magnetes potiſſimum ſpectabo. Magnetem ſcilicet ſum conſideraturus omni fere craſſitie carentem, in quo meatus arctiſſime uniti ab uno termino ad alterum ſecundum lineas rectas excurrant, atque adeo acui magneticæ perfecte ſimilem. Quare cum acus magnetica ratione virtutis a magnete non diſcrepet, nihil impediet, quominus in hoc negotio pro magnete acum magneticam ſubſtituam; quoniam in ea directio multo clarius percipi poteſt, quam in magnete plerumque difformi. Sit igitur hujusmodi acus magnetica etiam gravitatis expers, ſeu ita ſuſpenſa, ut ſeſe liberrime in eum ſitum, quem vortex terreſtris intendit, recipere poſſit. Sint igitur A & B duo terræ poli magnetici ſibi e diametro oppoſiti, in quorum altero A materia ſubtilis terræ meatus ingrediatur, in altero vero B iterum prorumpat. Quanquam autem difficile eſt determinatu, uter horum polorum boreæ vel auſtro reſpondeat, quoniam uterque eadem phænomena producit, tamen ne ambiguitas confuſionem pariat, citra errorem hoc dubium tollere licebit. Aſſumam itaque illum terræ polum A, ubi materia magnetica in meatus irrumpit, circa auſtrum eſſe poſitum, alterumque B materiam ſubtilem evomentem in regione boreali conſtitui. Erit ergo B polus terræ magneticus borealis, A vero auſtralis; neque enim hic plures

Acus magneticæ directio in vortice terreſtri indagatur.

Fig. 8.

D 3 polos

polos, etſi adſint, conſideraſſe opus eſt, cum quod de duobus demonſtrabitur, idem facile ad plures transferri queat. In figura porro interior circulus nucleum terræ, in quo vim magneticam maxime vigere probabile eſt, repræſentat, exterior vero ipſam terræ ſuperficiem, inter quam & nucleum cruſta terræ ſuperior continetur, ob plurima corpora heterogenea multo minore virtute prædita.

Acus magneticæ declinatio & inclinatio explicatur.

§. XXXVIII. Quanquam lineas illas curvas, per quas materia ſubtilis ex polo B egreſſa ad polum A revertitur, definire non auſim, tamen manifeſtum eſt motus directionem in ipſis polis A & B eſſe verticalem, in locis autem intermediis C & D horizontalem, quæ loca quaſi æquatorem magneticum exhibebunt: unde quo magis ad polos A & B accedatur, eo major proditura ſit inclinatio materiæ ſubtilis ad horizontem. Hæc autem generalis motus idea ad præſens inſtitutum ſufficiet, quoniam ſpecialis cognitio ob ſummam irregularitatem nequidem ſperari poteſt. Si igitur acus magnetica *ab*, in qua ſit *a* polus materiam magneticam deglutiens, *b* vero polus eructans, in loco quocunque vorticis terreſtris verſetur, ea ita diſponi debebit, ut ejus polus *a* materiæ ſubtili ingreſſum præbens polum terræ B naturæ contrariæ, unde materia ſubtilis advehitur, ſpectet, alter vero polus *b* ad terræ polum A vergat: eruntque adeo poli diverſæ naturæ ſibi amici, ejusdem autem naturæ inimici. Verſabitur ergo acus magnetica primum in circulo verticali per polos terræ magneticos transeunte, cujus a meridiano vero diſtantia in quovis terræ loco exhibebit declinationem magnetis. Præterea vero niſi acus verſetur in æquatore magnetico C D, ad horizontem inclinabitur, atque in hemiſphærio quidem boreali

CBD

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CBD acus terminus *a* boream ſpectans infra horizontem deprimetur, in altero autem hemiſphærio ſupra horizontem elevabitur, quæ inclinatio ad horizontem eo erit major, quo propius ad polos magneticos A vel B acus admoveatur. Quamobrem ex declinatione & inclinatione acus magneticæ in quovis loco terræ, directio curſus materiæ ſubtilis magneticæ cognoſci poterit.

§. XXXIX. Si igitur terra duos tantum haberet polos magneticos ſibi e diametro oppoſitos, tum ex factis aliquot obſervationibus non adeo difficile foret pro quovis terræ loco tam declinationem quam inclinationem acus magneticæ a priori aſſignare. Verum cum probabile ſit, terram quatuor polis magneticis gaudere, quorum bini neque ſint ſibi ediametro oppoſiti, neque a polis mundi æqualiter diſtent, tum multo magis arduum erit poſitionem acus magneticæ in quovis loco definire; quoniam comparatio virium, quibus verſus ſingulos polos urgetur, hincque reſultans media directio non niſi ſumma cum difficultate ſuſcipi poſſet. Primum enim ipſi poli magnetici ſine dubio diſſimili virtute ſunt præditi; unde fit ut etſi acus a binis æqualiter diſtet, tamen diſparibus viribus ad utrumque dirigatur. Tum vero, quia ipſa vis, quo propius ad quemque polum accedatur, ob majorem materiæ magneticæ copiam major evadit, hoc ipſum incrementum ac decrementum ante noſſe oporteret, quam quicquam concludi poſſet. Ob hunc ergo defectum nihil adhuc certi a priori definire licet: neque vero hoc ad theoriæ confirmationem quicquam conferret, cui unice me immorari debere ipſa quæſtionis propoſitio jubet. Ex his autem, quæ hactenus dicta ſunt, abunde ratio conſtat, tam declinationis & inclinationis acus magne-

Phænomena acus magneticæ a priori difficulter definiri poſſunt.

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magneticæ, quam harum ipſarum rerum mutabilitas, quæ ſucceſſu temporis ubique deprehenditur: quippe cujus cauſa in promotione ipſorum terræ polorum magneticorum apertiſſime eſt poſita.

Quomodo magnes a vortice ad motum cieri poſſit, docetur,

§. XL. Cum igitur phænomena acus magneticæ, atque adeo unius cujusque magnetis in ſe ſpectati, ſatis ſint explicata, quemadmodum duo pluresve magnetes inter ſe debeant eſſe affecti, per theoriam inveſtigabo. Præcipue itaque vorticem, qui circa quemlibet magnetem a materia ſubtili formatur, perpendere oportebit; hic autem vortex cum ad ſtatum permanentem fuerit perductus, ab æthere circumdato undique æqualiter comprimetur, hincque ipſe magnes undique æqualiter preſſus in quiete perſiſtet, ſi quidem jam eam directionem, quam affectat, ſit aſſecutus. Sit

Fig 9.

igitur magnes A B vortici in polo A ingreſſum, in B vero exitum præbens; atque materia ſubtilis erumpens circa β ad latera reflectetur, & ad α reverſa denuo in polum A ingredietur; in qua utraque reflexione materiæ ſubtilis æqualis utrinque vis impendetur, & ab hac æqualitate ſtatus æquilibrii pendebit. Quod ſi igitur eveniat, ut materia ſubtilis in β ſublatis obſtaculis directe progredi poſſit, tum ob ceſſantem hoc loco reflexionem vortex ex hac parte minori vi premetur, quam ad α, hincque ipſe magnes in A majori vi ſollicitatus quam in B, actu ſecundum directionem AB propelletur. Simili modo ſi ad α aliunde ſufficiens copia materiæ ſubtilis advehatur, ut reflexio materiæ vorticis in hac regione ſuſpendatur, tum preſſio vorticis ad β ſuperabit preſſionem ad α, ideoque ipſe magnes ad B majore vi impulſus ſecundum directionem BA promovebitur. Similiter intelligitur, ſi in β alius materiæ ſubtilis fluxus contra incur-

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incurrat, qui quaſi cum vortice ad B erumpente confligetur, tum ob majorem ad β reflexionem compreſſionem quoque vorticis in hoc loco prævalere debere; unde magnes ab altero illo materiæ ſubtilis incurſu repelletur in directione BA. Atque generatim, quibus in locis inflexio materiæ ſubtilis a cauſa externa vel augetur vel diminuitur, propter vim, quam inflexio poſtulat, vel auctam vel diminutam, compreſſio magnetis iisdem in locis vel major vel minor evadet; ſicque ſublato vorticis æquilibrio magnes ad motum ſollicitabitur.

Magnetes polis amicis ſe ſpectantes ſe mutuo attrahunt.

Fig. 10.

§. XLI. Ex his nunc facile omnes effectus, qui in magnetum mutua attractione ac repulſione cernuntur, determinari poterunt. Sint primo duo magnetes ita poſiti, ut eorum axes AB, & *ab* in directum jaceant, atque poli diverſi nominis A & *b* ſe mutuo reſpiciant. Hoc caſu perſpicuum eſt materiam magneticam ad *b* erumpentem non ſolum non tantam reſiſtentiam invenire, ſed etiam a vi elaſtica ætheris ad orificia alterius magnetis A urgeri; ita ut materia magnetica ad *b* proſiliens recta in meatus A irrumpat. Hinc igitur neque omnis materia ſubtilis ex *b* erumpens reflectetur, neque ea quæ ex B in A eſſet reverſura, curſum ſuum inflectet, ſed ulterius per δ & γ ad *a* usque progredietur; ita ut per confuſionem horum duorum vorticum, propemodum unicus vortex oriatur. Hanc ergo ob cauſam compreſſio utriusque vorticis inter polos A & *b* diminuetur, atque iſti ambo magnetes ad ſe mutuo urgebuntur. Permiſtio autem vorticum hincque compreſſionis ad A*b* diminutio a magnitudine utriusque vorticis & diſtantia magnetum pendebit; quo enim magnetes fuerint propiores, eo major pars materiæ magneticæ ad *b* erumpentis recta in A ingredietur, hincque eo minor portio refle-

reflectendo ad *a* perveniet: ob eandemque rationem eo minor portio materiæ ſubtilis in B exeuntis ad A reducetur: ſed maximam partem per γ & δ ad alterius magnetis polum *a* propagabitur. Manifeſtum ergo eſt, quo magnetes ſibi ſint viciniores, eo majori vi eos ad ſe invicem pelli debere; donec, ſi poli A & *b* ad mutuum contactum perveniant, omnis materia magnetica ad *b* effluens immediate in alterum magnetem ingrediatur: ſimulque omnis materia ad B erumpens in *a* usque reflectatur, ita ut unus vortex ſimplex exiſtat, niſi ambo magnetes magnitudine multum inter ſe diſcrepent.

Magnetes polis inimicis ſe ſpectantes, ſe mutuo modo attrahere modo repellere poſſunt.
Fig. 11.

§. XLII. Sint denuo axes duorum magnetum AB & *ab* in directum diſpoſiti, at poli ejusdem nominis ſeu inimici A & *a* ſe mutuo reſpiciant. Atque hoc caſu facile intelligitur ratione vorticum contrarium præcedentis caſus uſu venire oportere: cum enim materia ſubtilis ad B & *b* reflexa ſibi occurrat, & conflictus mutuus naſcatur, uterque vortex inter A & *a* propterea magis conprimetur, ex quo magnetes ſe mutuo repellere debebunt. Interim tamen, dum utraque materia ſubtilis ex B & ex *b* prorumpens motum ſuum, quantum fieri queat, conſervare conatur; dubium eſt nullum, quin pars quædam materiæ ex *b* redeuntis viam ſibi per alterum vorticem aperiat, atque per *b* in A ingrediatur; ſimilique modo vorticis ex B recurrentis portio per *f* ad *a* penetrabit, ſicque permiſtio quædam vorticum modo major modo minor orietur. Ex tali ergo vorticum permiſtione ob rationem ante allegatam attractio oriri debebit; quatenus autem reliquæ vorticum portiones eo promtius in *g* & *h* inflectuntur, ex hac fortiori reflexione repulſio naſcetur, qui duo effectus prout alter alterum ſuperaverit,

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vel attractionem vel repulſionem generabunt. Quod ſi ergo duo magnetes hoc modo diſponantur, fieri poterit, ut ſe mutuo æque attrahant, ac repellant; quin etiam accidere poteſt, ut variata magnetum diſtantia modo attractio modo repulſio prævaleat, quæ anomaliæ a figura utriusque magnetis plurimum pendebunt; ac præterea in ſe tam erunt inconſtantes, ut repetito eodem plane caſu, raro eadem phænomena obſerventur. Tam egregie autem hæc cum experimentis, quæ Celeb. Muſchenbroeckio debemus, conſentiunt, ut vel ex hoc ſolo conſenſu theoria mea maximum adipiſcatur firmamentum; propterea quod ii ipſi effectus, qui iſti Auctori plane inextricabiles, atque cauſam quaſi intelligentem arguere videbantur, tam prono alveo ex iſto fonte derivantur.

§. XLIII. Hi ergo duo magnetes in iſto ſtatu præternaturali perſeverare non poterunt, niſi vi in eo detineantur. Ponamus igitur utrumque magnetem inſtar verſorii mobilem eſſe ſuper cuſpide verticali centrum gravitatis C & c ſuſtinente. In hoc ſtatu, ſi mentem a directione generali a vortice terræ orta abſtrahamus, ratio utique nulla foret, cur hi magnetes ſe in hanc potius quam aliam plagam converterent. At propter permixtionem vorticum, quam figura repræſentat, polus A verſus *g*, polus *a* vero verſus *h* recedere debebit; hocque modo poli cognomines ſimulque inimici A & *a* ſe mutuo fugare videbuntur. Hic nimirum ſimilis effectus oriri debet, quo meatum magneticum a vortice terræ dirigi eſt oſtenſum: materia namque magnetica per *e* veniens oblique in meatus magneticos AB incurrit, hosque idcirco ſecundum eam ipſam directionem, in qua materia ſubtilis fertur, diſponere conatur. Hinc magnes AB circa C ver- Quomodo duo magnetes ſe fugent oſtenditur.

verſus *g* & alter circa *c* verſus *h* converti debebit; ſiquidem permiſtio vorticum eo fuerit facta modo, quem figura exhibet; ſin autem permiſtio contrario modo contigerit, tum etiam magnetes in plagas contrarias a ſe invicem recedent. Quousque autem poli A & *a* a ſe invicem removeantur in ſequenti §. videbimus; interim hoc loco tum ratione permiſtionis vorticum, tum ratione converſionis plurimum referre notandum eſt, quomodo diſpoſitio magnetum horum ſe habeat, reſpectu plagarum mundi; vis enim dirigens vorticis terreſtris maxime permiſtionem horum vorticum juvabit, atque efficiet, ut magnetes verſus hanc potius quam alteram plagam convertantur. Hanc autem circumſtantiam ſitus magni eſſe momenti experimenta ſatis comprobant.

Quomodo duo magnetes ſuſpenſi in ſe mutuo agant.

§. XLIV. Ponamus utrumque magnetem caſus præcedentis jam per angulum rectum a ſe invicem receſſiſſe, ſeu duos magnetes AB & *ab* ſuper cuſpidibus C & *c* mobiles in eo ſitu eſſe poſitos, quem figura repræſentat; ubi ſcilicet ſint axes utriusque magnetis inter ſe paralleli, ſed ita ut poli diverſi nominis A & *b*, itemque B & *a* in eandem plagam ſpectent. Primum quidem vorticem terreſtrem cogitatione tollamus, atque manifeſtum portionem materiæ ſubtilis in B profilientis ad alterius magnetis polum *a* eſſe tranſituram; ſimiliterque portionem materiæ ſubtilis in *b* erumpentis in alterius magnetis polum A eſſe ingreſſuram. Hæc ſcilicet deflexio a tramite conſueto ideo fiet, quia hoc modo ſubitanea reflexio evitatur; ex quo ſimul perſpicitur, ſi hi magnetes ſibi nimis eſſet vicini, tum hujusmodi communicationem nil lucri fore allaturam. In caſu autem, quem figura repræſentat, cum utrinque omnia ſint ſimilia ratio erit nulla, cur magnetes convertantur. Quodſi vero alteruter

Fig. 12.

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vel tantillum declinetur, fient ex una parte poli amici ſibi propiores quam in altera parte, & hancobrem ſe mutuo attrahendo ſeſe in ſitum fig. 10 repræſentatum component; niſi inter ſe nimis ſint propinqui. Sin autem effectus vorticis terræ inſuper accedat, tum ex combinatione cauſarum effectus dijudicari debebit: ubi quidem perſpicuum eſt, quo remotiores fuerint magnetes, eo majorem eſſe futurum effectum a vortice terreſtri oriundum, in exiguis autem diſtantiis vires magnetum eſſe prævalituras.

§. XLV. Diſtincte igitur mihi expoſuiſſe videor, quemadmodum duo magnetes tam ratione attractionis & repulſionis, quam ratione directionis in ſe mutuo agant: qui effectus quo magis ſunt ſingulares & admirabiles, eo fortius cauſam hic a me aſſignatam, unde tam plane & luculenter conſequuntur, confirmant. Cum enim effectus ſimplices plerumque a pluribus cauſis oriri poſſint, atque difficile ſit ex his, quæ vera ſit, diſcernere: ſic effectus compoſiti maximeque complicati, cujusmodi ſunt phænomena magnetis, nonniſi ab unica cauſa proficiſci poſſunt, a qua, ſi vel tantillum aberremus, plerorumque phænomenorum rationem reddere haud valeamus. Quamobrem cum præcipuorum magnetis effectuum cauſa tam naturaliter in theoria hic expoſita contineatur, dubitari profecto nequit, quin hæc ſola theoria cum veritate conſentiat. In hoc autem genere nullum adhuc inveni experimentum, cujus eventum non ſolum non theoriæ meæ maxime congruum deprehenderim, ſed etiam a priori facile prædixerim. Quoniam vero ea experimenta, quæ circa actionem duorum magnetum in ſe mutuo ſunt inſtituta, non difficulter ad caſus evolutos reducuntur, iis ſingulis explicandis non immorabor; cum eorum ſolutio

Theoria ex conſenſu cum experientia magis confirmatur.

non

non ſolum ex jam allatis manifeſto ſequatur, ſed etiam tædioſum foret in ejusmodi rebus, in quibus ne ſcrupulus quidem ſuboriri queat, longius inhærere. Progrediar itaque ad alius generis phænomena, poſtquam hoc unicum monuero, corpora quæcunque ferro excepto inter binos magnetes interpoſita, eorum actionem mutuam nullo modo turbare; cujus circumſtantiæ, etſi ea Celeb. Muſſchenbroeckio maximam difficultatem habere viſa eſt, ratio ex ſuperioribus in promtu eſt, cum ſtatuiſſem materiam magneticam cuncta corpora, præter ferrum ac magnetem liberrime permeare: quæ aſſumtio cum ſit naturæ maxime conformis, qua conſtat omnia corpora ætheri aliisque fluidis ſubtilibus, uti calori, eſſe pervia, tum etiam ex his ipſis experimentis firmiſſime corroboratur.

Virtutis magneticæ communicatio cum ferro perpenditur.

§. XLVI. Jam præter attractionem, repulſionem ac directionem, in magnete potiſſimum notatu digna eſt ejus vis communicativa, qua cum ferro & chalybe ſimilem ſui vim ita communicat, ut de ſua vi propria nihil amittat: quæ proprietas eorum animos, qui in cauſam phænomenorum magnetis inquiſiverunt, maxime torſit. Ex cognita autem vera ſtructura, qua virtus magnetica continetur, manifeſtum eſt, ad virtutem magneticam cuiquam corpori conciliandam nil aliud requiri, niſi ut in eo meatus magnetici efformentur, qui continuo tractu ab uno corporis termino ad alterum progrediantur. Ad hoc ergo primum in corpore ineſſe debent ejusmodi pori minimi ſubtiliſſimis filamentis obſiti & quaſi valvulis inſtructi; tum autem hi pori ita disponi debent, ut canales continuos conſtituant, & valvulæ per totum tractum in eundem ſenſum ſpectent. Hujusmodi autem poris præter magnetem præditum eſt ferrum, et

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et chalybs cum aliis corporibus, in quibus hæ ipſæ materiæ abundant. Hæc corpora etiamſi ejusmodi poris ſcateant, tamen, niſi pori in meatus magneticos ſint diſpoſiti, virtutis magneticæ erunt expertia. Quamobrem ut his virtus magnetica inducatur, primum neceſſe eſt, ut pori ſint mobiles æque ac filamenta, quibus ſunt obſiti; tum vero ejusmodi vis accedere debebit, quæ poros in debitum ordinem redigere valeat. In poris autem non opus eſt, ut ſenſibilis mobilitas ſtatuatur, quoniam quæque corpora poris maxime ſunt plena, ac leviſſima mutatio ſufficere poteſt, ad poros in ſeriem continuam diſponendos. Filamenta vero, quæ in poris valvularum vices gerunt, ſine mobilitate concipi vix poſſunt: interim tamenper ſe perſpicuum eſt, hanc mobilitatem in aliis corporibus eſſe poſſe majorem in aliis minorem.

Quomodo in ferro vis magnetica generetur.

§, XLVII. In ferro ergo vel chalybe virtute magnetica nondum imbuto pori iſti nullo certo ordine erunt diſpoſiti; ſed ita confuſe diſperſi, ut neque meatus continuos conſtituant, neque valvulæ ex filamentis formatæ ad eandem plagam reſpiciant; quo in ſtatu quam diu hæc corpora manebunt, nulla prorſus vi magnetica pollebunt, ſed inſtar reliquorum corporum inertia jacebunt. Neque vero etiam æther quieſcens circumfuſus in his corporibus certos meatus efformare poſſet, quod undique eadem vi premeret, hincque poros, etiamſi ſint mobiles, in unam potius plagam quam aliam diſponere non valeret. At ſi æther, uti in vortice terreſtri uſu venit, jam ſit in motu, tum ſecundum motus ſui directionem majori vi in poros extimos intrabit, qua tandem poros extremos ſecundum directionem ſuam ordinabit: cum enim fluxus materiæ magneticæ integros magnetes dirigere valeat, multo magis minimas moleculas, cujusmodi ſunt

fila-

ſilamenta, ciere valebit. Statim autem ac materia magnetica in uno loco per extremos poros penetraverit, ob ingentem quam jam acquiſivit celeritatem, facilius poros internos in debitum ordinem diſponet, dummodo hujusmodi pori reperiantur continui. Sin autem materia magnetica jam ad alterum usque terminum penetraverit, tum ibi prorumpens vorticem formabit, prorſus uti circa ſingulos magnetes exiſtere monſtravi: quo formato ferrum ſeu chalybs pari virtute magnetica pollebit, atque ipſe magnes; ſin autem pori magnetici non continuo tractu procedant, ſed ſpatia peregrinâ materiâ repleta interjaceant, uti ſit in mineris ferri non admodum fœcundis, tum ejusmodi corpora vi magnetica impraegnari non poterunt.

Ferrum ſucceſſu temporis tandem virtute magnetica impraegnatur.

§. XLVIII. Ex his igitur intelligitur, a vortice magnetico terram ambiente in ferro tandem vim magneticam generari poſſe; cum iſte vortex ad hunc effectum producendum perpetuo vires ſuas exerceat. Quoniam vero in vortice terreſtri materia magnetica & admodum lente movetur, nec in ſatis magna copia cum æthere eſt permixta, ejus vis perquam erit debilis; unde quo hæc vis fuerit minor, eo plus temporis requiretur ad virtutem ſenſibilem in ferro excitandam. Ita videmus poſt longum demum temporis intervallum bacillos ferreos virtute magneticâ impraegnari. Neque vero iſte effectus quovis modo obtinetur, verum neceſſe eſt ut ejusmodi bacilli ferrei conſtanter in eodem ſitu retineantur. Niſi enim idem perpetuo ſitus conſervetur, materia magnetica ſecundum aliam directionem ageret, ideoque effectum jam ante productum rurſus deſtrueret. Praeterea vero eo longiore temporis intervallo opus erit, quo minus mobiles erunt pori in ferro contenti, & quo rigidiora fuerint ſilamenta, ut non tam facile in ſitum convenientem recli-

reckinari queant. Ex quo intelligitur, ſi qua cauſa internæ ferri particulæ mobiliores reddantur, eo citius ferrum virtute magnetica imprægnatum iri. Hinc ferrum calidum ſi in eodem ſitu diu reponatur, virtutem magneticam citius acquiret, quam frigidum; quoniam per calorem partes ferri rareſiunt, atque facilius inter ſe commoventur. Non parum quoque ad imprægnationem accelerandam conferre poterunt mallei ictus vel limæ attritus, quippe quibus introitus materiæ ſubtilis, atque pororum commotio non mediocriter adjuvatur. Quin etiam diſſolutione ferri lexta, cujusmodi evenit, ſi ferrum lapidi infixum injuriæ tempeſtatis diu fuerit expoſitum, ac ferri particulæ ſolutæ ſenſim ſe in poros lapidis inſinuent, actioni vorticis magnetici liberior aditus aperitur, atque dum ſingulæ ferri particulæ debito modo diſponuntur, lapis tandem iis repletus perfectum magnetem mentietur.

Virtus magnetica in ferro facile iterum deſtrui poteſt.

§. XLIX. Vis magnetica hoc modo in ferrum illata eo erit fortior, quo plures meatus continuos materia magnetica ſibi efformaverit: ſimul autem hæc vis in ferro eo pertinacius inhærebit, quo id fuerit durius, ſive ab initio jam tale fuerit, ſive demum poſtquam virtutem magneticam jam eſt adeptum, tale ſit factum. Quo durius ſcilicet fuerit ferrum, eo difficilius quidem virtutem magneticam adipiſcitur, at vero adeptam pertinacius conſervat; contra vero ferrum mollius promtius quidem virtute magnetica imbuitur, ſed eo facilius ac citius eam iterum amittit. Hanc ob cauſam videmus ferrum multo facilius virtutem magneticam impetrare quam chalybem; chalybem vero vim ſemel receptam fortius retinere. Ferrum autem virtute magnetica imbutum iterum privatur, ſi in ſitu contrario vortici terreſtri

 diu

diu exponatur; quo ſit ut materia ſubtilis ſibi ſecundum aliam directionem meatus formando eos, qui jam ante inerant, rurſus deſtruat. Hæc ergo deſtructio iiſdem mediis promoveri poteſt, quibus ante impraegnationem accelerari oſtendimus. Inflexio nimirum, mallei ictus, atque limatio, quibus operationibus pororum ordo turbari, & meatus interrumpi poſſunt, virtutem magneticam afficient, eamque penitus delere valebunt. Imprimis autem calor atque ignitio plurimum virtutem magneticam turbabunt, atque etiam ob hanc rationem ipſe magnes vi ſua penitus exui poterit; quæ omnia cum experientia tam arcte conſentiunt, ut etiam ex hac parte nullum dubium circa veritatem hujus theoriæ ſupereſſe poſſit.

Figura ferri plurimum confert, ad vim magneticam accipiendam.

§. L. Quo autem ferri fruſtum virtute magnetica imbuatur, plurimum intereſt, cujusmodi habeat figuram, & in quonam ſitu hæc figura reſpectu vorticis materiæ magneticæ reponatur. Quod ad figuram attinet, aptiſſima deprehenditur ea, quæ ſit oblonga inſtar trabeculæ efformata, neque nimis tenuis neque nimis craſſa. Quarum conditionum ratio ex theoria dilucide reddi poteſt. Primum

Fig. 13.

enim patet figuram rectam AB præſtare incurvatæ *ab*, eo quod materia ſubtilis ſemel in A vel *a* ingreſſa curſum ſuum facilius in directum proſequitur, quam ſecundum lineam curvam vel inflexam. Quoniam deinde materia ſubtilis magnetica per poros ferri facillime movetur, maximam autem reſiſtentiam offendit, dum ex ferro in ætherem apertum egreditur, in trabecula recta AB parum materiæ magneticæ ad latera effluet, ſed fere omnis ſecundum longitudinem AB penetrabit, ſicque meatus rectos ſecundum longitudinem AB extenſos & inter ſe parallelos formabit, quibus

bus fortiſſima virtus magnetica efficitur. In trabecula autem curvilinea *ab* meatus incurvati non tam facile formabuntur, ſed in parte convexa non exigua materiæ ſubtilis portio in ætherem prorumpet, atque vim magneticam debilitabit. Quo autem meatus inter ſe paralleli producantur, craſſities bacilli ſatis exigua eſſe debet, ne ulla divergentia, qua curſus meatuum perturbetur, locum habere queat: interim tamen nimia gracilitas nocebit impraegnationi, propterea quod meatuum numerus diminuitur, atque materia ad latera effluens plurimum de virtute auferet. Hoc idem incommodum ſe offeret in bacillis ſeu virgis nimis longis, in quibus fere omnis materia ſubtilis ad latera exire poteſt, antequam ad alteram extremitatem deferatur. Virga autem nimis brevis, etiamſi his incommodis careat, tamen hoc vitio laborat, quod in tam exiguo ſpatio vortex materiæ ob viam percurrendam nimis curvam formari nequeat. Dabitur ergo & longitudo & craſſities bacillo ferreo tribuenda, quæ ſit aptiſſima ad virtutem magneticam accipiendam, omnino uti experientia teſtatur.

Fruſtum ferri nimis craſſum ineptum eſt ad magnetismum. Fig. 14.

§ LI. Quod autem figura nimis craſſa inepta ſit ad vim magneticam recipiendam, ex theoria facile perſpicitur. In hujusmodi enim figura ferri AB, materia ſubtilis ad A ingreſſa facile a tramite recto deflectet. Quamvis enim conetur in directum progredi, tamen ſi ob inæqualitatem particularem ferri, hinc inde ad latera minorem reſiſtentiam inveniat, eo deviabit uti in *e*, hocque modo non ſolum motus rectilineus turbabitur, ſed etiam motus materiæ ſubtilis in *a* ingreſſæ ac fortaſſe in directum progreſſuræ præpedietur. Imprimis autem hoc incommodum in altero termino B cernetur, ad quem ſi que materia ſubtilis penetraverit, tamen

potius ad latera in *f* deflectet, ubi minorem invenit reſiſtentiam, quam ſi recta erumperet. Hoc igitur modo materia ſubtilis ſibi ipſa eſt obſtaculo in fruſto ferreo nimis craſſo, atque impedit, quominus meatus regulares formari queant. Evadet quoque ingens portio materiæ ſubtilis per latera ferri, atque viciſſim ad latera nova materia ſubtilis ingredietur, quibus omnibus fit, ut vortex magneticus circa hujusmodi corpora vix ac ne vix quidem formari poſſit. Non parum tamen in hujusmodi corporibus virtus magnetica augeri poteſt, ſi utrinque in A & B acuminentur; hoc enim modo materia ſubtilis, quæ ante in exitu ad *f* deflectebatur, nunc ad cuſpitem delata curſum ad latera inflectere non poterit, ſed recta in ætherem prorumpet, vorticemque facilius producet. Experientia autem docet, bacillos ferreos utrinque in cuspides deſinentes multo majorem acquirere virtutem magneticam, quam ſi terminos habeant obtuſos; quo certe phænomeno fluxus materiæ cujusdam ſubtilis per meatus ferri extra omne dubium collocatur.

Imprægnatio vis magneticæ a ſitu bacilli ferrei maxime pendet.

§. LII. Bacillus ergo ferreus, ſi in vortice terreſtri ita collocetur, ut ejus longitudo cum directione materiæ ſubtilis congruat, eo facilius virtute magnetica imprægnabitur, quo magis apta fuerit ejus figura ad hanc virtutem recipiendam. Quodſi autem bacilli directio non multum diſcrepet a directione motus materiæ ſubtilis in vortice, virtutem magneticam quidem etiam acquiret, at cum tardius tum debiliorem; quæ difficultas imprægnationis eo erit major, quo magis directio bacilli a directione vorticis differat; ac ſi bacillus ad hanc vorticis directionem normaliter conſtituatur, tum nullam plane unquam vim recipere poterit, quia materia ſubtilis æqua vi utrinque ingredi conabitur. Detineatur hujusmodi bacillus AB in noſtris quidem regio-

Fig. 15.

regionibus in ſitu verticali, ubi directio materiæ ſubtilis *a*A, B*b* cum horizonte facit angulum circiter 60°, cum bacillo ergo angulum 30°. Cum igitur uti aſſumſi materia ſubtilis ex terræ polo magnetico boreali erumpat, ea in directione *a*A ad bacillum perveniet, tandemque in A ſibi ingreſſum aperiet: ſtatim autem directionem ſuam in bacillo inflectet ſecundum ipſius longitudinem, egreſſum ad latera evitatura; ſicque tandem formatis meatibus ſecundum longitudinem AB per bacillum transfluet, vorticemque peculiarem generabit, qui ad A in bacillum ingredietur, ad B vero egredietur. Hinc ergo bacillus in magnetem transformabitur, polos ſuos ad A & B habentem, quorum illo A, ſi ſibi relinquatur, polum terræ magneticum borealem, altero vero B auſtralem reſpiciet. Quamvis ſcilicet hic bacillus in ſitu verticali detentus virtutem magneticam ſit conſecutus, tamen ſibi relictus non amplius hanc poſitionem affectabit, ſed ſecundum curſum vorticis magnetici, quocunque terrarum transferatur, ſeſe diſponet. Sub æquatore igitur terræ magnetico idem bacillus verticaliter fixus nunquam virtutem magneticam accipiet, quia ibi fluxus materiæ ſubtilis ſit ſecundum directionem horizontalem, reſpectu bacilli indifferentem.

§. LIII. Quæ ferro in vortice terræ verſanti accidere docui, eadem efficientur in ferro, quod in vortice cujusque magnetis eſt poſitum, ſimili quidem modo, at multo citius ac promtius; quoniam materia ſubtilis magnetica in vortice magnetem ambiente cum majori copia adeſt, tum etiam motu incitatiori circumfertur. Hinc fit, ut non ſolum virtus magnetica cum ferro citius communicetur, ſed etiam figura ferri minus idonea non tantopere efformatio- *Magnes ferrum attrahet.*

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nem meatuum magneticorum impediat. Quo enim vis materiæ magneticæ eſt major, eo facilius obſtacula in ferro ſuperabit, atque vorticem magneticum conſtituet. Hinc quævis ferri fruſta in vicinia magnetis poſita ſtatim a materia magnetica permeantur, ac propterea inſtar magnetis attrahentur. Repulſio autem in ferro non tam facile locum invenit, quia vortex ſtatim ac circa ferrum generatur, ſimul ita diſponitur, ut ejus motus cum motu vorticis magnetis conſentiat, ideoque vorticum confuſio oriatur; unde poli amici ſe mutuo reſpicient, atque fortiſſima attractio gignetur. Sunt enim pori minimæque ferri particulæ admodum mobiles, & a vi magnetica ſufficiente facillime in quavis directione ad meatus magneticos formandos diſponuntur: unde fit, ut ferrum virtute magnetica jam imbutum, etiamſi in ſitum contrarium reſpectu magnetis collocetur, in quo repelli deberet, tamen mox vim priorem amittat, ejusque loco vorticem in contrariam plagam directum accipiat, qui cum vortice magnetis conſentiat, atque attractionem producat. Magnes autem multo pertinacius vorticem ſuum retinet, qui ab alio magnete vix de ſtatu ſuo turbari queat: cujus phænomeni ratio eſt, quod meatus in magnete formati minus ſint mobiles, & in ſitu ſuo multo fortius perſiſtant. Chalybs autem, qui eſt ferrum magis induratum, medium quendam locum inter magnetem ac ferrum tenet, ita ut difficilius virtutem magneticam acquirat quam ferrum, acquiſitam autem firmius retineat; unde ratio patet, cur magnes ferrum facilius attrahat, quam chalybem: contra autem chalybs vim magneticam diutius conſervet quam ferrum. Atque ob hanc rationem acus magneticas ex chalybe duriſſimo fabricari præſtat, quam ex ferro.

§. LIV.

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§. LIV. Inſtitutum eſt nuper coram Academia Scient; Petropolitana experimentun, quod ob eventum prorſus ſingularem ac theoriam meam mirifice confirmantem hic prætermittere non poſſum. Fecit ibi Expertus Chymicus Cel. Gellert peculiari artificio mixturam ex ferro & ſtanno, quam magnetes generoſi ac fortes prorſus non attraxerunt, magnes vero exiguus & debilis attractam tenebat. Erant ergo in hac mixtura particulæ ferri ita cum particulis ſtanni permixtæ, ut meatus continui formari non poſſent; ſed materia ſubtilis, poſtquam per ſingulas ferri particulas tranſiiſſet, in particulis ſtanni ob reſiſtentiam ætheris, motum ſuum quaſi ſingulis momentis amitteret. Ferri autem particulæ nimis erant parvæ, quam ut circa ſingulas vortex magneticus generaretur. Hanc ob cauſam materia ſubtilis celeritate non nimis magna mixturam permeare potuit. Quo autem corpus a magnete attrahatur, neceſſe eſt ut circa id vortex materiæ magneticæ generetur; atque ad vorticem formandum requiritur, ut materia ſubtilis per viſcera corporis multo celerius promoveatur, quam extra, tantaque celeritate erumpat, ut ab æthere externo reflectatur. Cum igitur in vortice magnetis generoſi materia ſubtilis jam ingenti velocitate moveatur, fieri poteſt ut hæc celeritas non ſit minor, quam ea, qua materia ſubtilis per mixturam tranſierit: & hanc ob rem circa mixturam neque vortex generari, neque proinde mixtura a magnete generoſo attrahi potuit. Circa magnetem autem debilem materia ſubtilis multo minori celeritate gyratur, quæ adeo ſi multo minor fuerit, quam celeritas materiæ ſubtilis mixturam permeantis, motus vorticoſus oriri, hincque attractio evenire potuit. Oportet autem ad hoc experimentum ferrum cum ſtanno intime permiſceri, ut materia ſubtilis per hanc materiam heterogeneam

Singulare phænomenon magnetis explicatur.

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terogeneam motu quaſi uniformi progrediatur. Singulare omnino ac novum eſt hoc experimentum, quo conſtat a magnete debiliori effectum produci poſſe, cui producendo magnes generoſus ſit impar.

Quomodo ferro per magnetem virtus magnetica inducatur.

§. LV. Quoniam vidimus cum ferro eo facilius vim magneticam communicari, quo copioſior motuque incitatior fuerit materia magnetica, manifeſtum eſt, quo propius ferrum ad magnetem admoveatur, eo citius ac fortius id virtute magnetica imbui debere. Maxime autem virtutis communicatio ac proinde attractio ſeſe exerit in polis magnetis, ubi materia ſubtilis copioſiſſime vel influit in magnetem vel ex eo prorumpit, hocque majori celeritate, quam in ullo alio vorticis loco. Quæ ut clarius perſpiciantur, ſit AB magnes, cujus polus materiæ ſubtili introitum præbens ſit A, alter ubi materia ſubtilis erumpit, B. Quodſi jam ad polum B acus ferrea vel aliud fruſtulum *ab* reponatur, mox portio materiæ ſubtilis ad B erumpentis in *a* irruet, & quia tum celerrime movetur tum in magna copia adeſt, in acu ſecundum motus ſui directionem meatus efformabit, atque in *b* iterum exiens ab æthere ambiente reflectetur; a qua reflexione acus ad magnetem urgebitur. Transmutabitur ergo acus ratione virtutis ita in magnetem, ut poli B & *a* ſibi fiant amici. Idem eveniet ſi acus *ab* ad alterum magnetis polum A ponatur, tum enim non ſolum in ejus termino remotiori *a* a vortice magnetis materia ſubtilis intrabit, ſed etiam, quia pori jam ſunt materia hac ſubtili ſummopere elaſtica repleti, ea in *b* propter reſiſtentiam ætheris ſublatam erumpet, ſicque poſt ſe per *a* continuum fluxum materiæ ſubtilis trahet; quocirca eadem plane phænomena in acu polo A admota obſervari debebunt, quæ

Fig. 15.

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quæ in acu polo B admota deprehenduntur; hincque difficulter natura polorum magnetis, uter materiam ſubtilem vel abſorbeat, vel emittat, per experientiam diſtingui poterit.

§. LVI. Ferrum ergo in vicinia magnetis virtute magnetica imbuitur, ita ut non ſolum ad magnetem attrahatur; ſed etiam ipſum in alia ferri fruſtula vim exerere valeat. Ejus quidem virtus a magnete plurimum ſuſtinetur, unde continuo materia magnetica ingenti copia advehitur: quo fit, ut etiamſi non adeo multi meatus magnetici in ferro ſint formati, tamen ob abundantiam materiæ magneticæ virtus ſit perquam conſpicua. Statim autem ac ferrum a magnete removetur, ejus vis vehementer debilitabitur; eo quod primum a vortice magnetis non amplius adjuvatur, tum vero materia magnetica in multo minori copia in meatus efformatos ingreditur. Scilicet eo debilior erit vis in ferrum translata, quo pauciores meatus fuerint formati, atque ob eandem rationem hæc ipſa vis eo citius peribit: imprimis autem plurimum refert, cujusmodi figura ferrum ſit præditum. Hinc intelligitur vim cum ferro communicatam eo fore majorem ac ſtabiliorem, quo plures meatus in eo formentur curſu parallelo in directum extenſi. Perſpicuum igitur eſt, quo propius ferrum ad magnetem admoveatur, ob majorem materiæ copiam, eo plures meatus efformari debere: quocirca in ipſo contactu imprægnatio erit maxima, quia materia magnetica non ſolum pura in ferrum ex magnete irrumpit, ſed etiam ea ipſa fere celeritate, qua in meatibus magnetis movetur. Imprægnatio porro per attritum maxime corroboratur; attritu enim partim contactus arctior efficitur, partim minimæ particulæ ferri ſuccutiuntur, quo facilius ſe in ordinem debitum diſponere queant.

Quomodo per attritum vis magneti maxime communicetur.

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Virtus magnetica ad inſignem diſtantiam ex tendi poteſt.

Fig. 17.

§. LVII. Quanquam vortex magnetem ambiens ad magnam diſtantiam extenditur, quoad penitus ceſſet, atque eo longius pertingat, quo major ac nobilior fuerit magnes; tamen ope ferri, virtus magnetis ad multo majorem diſtantiam expandi poteſt. Sit enim magnes A B, cujus vortex non ultra *e f* pertingat; ſi ad ejus polum B admoveatur bacillus ferreus *ab*, portio materiæ magneticæ per hunc bacillum ad *b* usque deducetur, unde ad A revertens vorticem ad *g h* usque pertingentem exhibebit, debiliorem quidem quam erat ad *e f*. Quod ſi porro ad *b* alius bacillus ferreus $\alpha\beta$ teneatur, per hunc portio materiæ magneticæ ad *b* erumpentis ultra β ad *i k* propagabitur, ſicque vortex magnetis usque ad *i k* pertingere videbitur, cum demtis bacillis non ultra *ef* extendatur. Sic itaque ferrum in vicinia magnetis ipſum vi magnetica pollebit, eamque ulterius extendere valebit. Ob hanc rationem duo fruſta ferrea in vicinia magnetis non ſolum ſe mutuo attrahent, ſed etiamſi a magnete ad ſatis notabilem diſtantiam removeantur, tamen attractio mutua non ceſſabit; perinde ac ſi vorticem magnetis ſecum diducerent & dilatarent. Sin autem ferri maſſa ſtatim ab initio in tanta a magnete diſtantia teneatur, nulla vis ipſi ineſſe deprehendetur, non tam quod ipſi nondum inducta ſit virtus magnetica; ſed præcipue quod vortex magnetis non ad tantam diſtantiam porrigatur, ad quam tamen per ſucceſſivam ferri a magnete remotionem diſtendi poſſet.

Vortex magneticus ad oculum demonſtrari poteſt.

§. LVIII. Quo minus autem exiſtentia vorticis quemque magnetem ambientis in dubium vocari poſſit, non ſolum ipſe vortex, ſed etiam curſus materiæ ſubtilis in quovis loco circa magnetem oculis ſpectandus exhiberi poteſt.

Etſi

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Etſi enim notiſſimum eſt experimentum, quo magnes limatura ferri circumfundi ſolet; tamen poſitio particularum ferri tam copiam quam directionem materiæ ſubtilis vorticem conſtituentis evidenter demonſtrat. Cum enim particulæ ferri, quibus limatura conſtat, non ſint globoſæ, ſed tenues inſtar acicularum formatæ, eæ non ſolum attrahentur, ſed etiam ſecundum longitudinem in eam ipſam directionem diſponentur, in qua materia vorticis movetur. Supra enim eſt oſtenſum acum ferream in vortice magnetis poſitam ſecundum directionem motus materiæ ſubtilis ſeſe diſponere debere. Hinc limatura ferri magneti circumfuſa primum diſtincte oſtendet polos magnetis, ubi materia magnetica vel ingreditur vel egreditur; tum vero copia ejus in quovis loco copiam materiæ ſubtilis declarabit. Quod ſi ergo per appropinquationem alius magnetis vel ferri ſtatus vorticis turbatur vel immutatur, inſpectio limaturæ ferri circumfuſæ hanc mutationem ſtatim indicabit. Atque hoc pacto confuſio atque alteratio vorticum, quam ſupra in variis circumſtantiis evenire debere docui, per experientiam oculis cerni, ſicque firmiſſime demonſtrari poterit. Jucundum ſane hoc modo ſpectaculum ſenſibus offertur, quo materiæ prorſus inviſibilis motum ac directionem tam diſtincte percipere licet: theoriæ autem meæ aliunde quidem ſatis ſuperque confirmatæ veritas per hujusmodi experimenta magna cum voluptate agnoſcetur, & comprobabitur.

§. LIX. Quantum armatura ad multiplicandam cujusque magnetis virtutem conferat, ex hac theoria quoque evidenter demonſtrari poteſt. Sit enim magnes AB polos habens in A & B, per quorum alterum A materia ſubtilis in- *Vis magnetis per armaturam intenditur.* *Fig. 18.*

ingrediatur, per alterum B egrediatur: atque uti in armatura fieri ſolet, utrinque ad polos A & B laminæ ferreæ arctiſſime coaptentur, quæ in altera extremitate adjunctos habeant pedunculos *a* & *b* magneti firmiſſime adjacentes, ut armatura cum magnete quaſi unum corpus continuum conſtituat. In hoc ſtatu materia ſubtilis magnetica ad B exitura in lamina curſum ſuum verſus pedunculum *b* inflectet, eo quod longe minorem invenit reſiſtentiam in ferro, motu etiam inflexo progrediendi, quam directe in ætherem erumpendi. Maxima ergo materiæ ſubtilis portio ad pedunculum delata in ejus baſi *b* prorumpet, huncque adeo in locum exiguum polus magnetis, qui antea per totam baſin B erat diffuſus, coarctabitur; ex quo vis attractiva hujus pedunculi tanto major evadet. Idem eveniet in altero polo A, in quem materia magnetica in pedunculo *a* & lamina contenta facilius irruet, ſicque motus materiæ ſubtilis mox ita immutabitur, ut ex *b* erumpens per *c* curſum inflectendo in pedunculum *a* ingrediatur, hincque per magnetem ad *b* revertendo vorticem perennem conſtituat. Qui vortex, cum in unicam fere regionem per *c* cogatur, cum ante circa univerſum magnetem eſſet diffuſus, nunc multo majori gaudebit vi tam attractiva quam directrice, omnino uti experientia clariſſime demonſtrat.

Quomodo vis amborum magnetis polorum per armaturam uniatur. Fig. 19.

§. LX. Effectus autem magnetis hoc modo armati ideo quoque erit major, quoniam ambo poli *a* & *b* in geſtandis oneribus ſe mutuo adjuvabunt, hincque vim quaſi duplicabunt, niſi forte onera ferrea ita applicentur, ut vires amborum polorum ſe mutuo deſtruant. Hoc autem modo vis unietur, ſi fruſtum ferri idoneæ figuræ EF ad baſes pedunculorum *a* & *b* ita applicetur, ut contactus ſit perfectus.

Tum

Tum enim materia ſubtilis ad *b* effluens per hoc ferramentum viam ſibi aperiet, atque curſu ad C *c* reflexo ad *a* in magnetem revertetur, hocque modo ferrum ad utrumque pedunculum ſimul attrahetur; quæ adeo vis longiſſime ſuperabit eam, quam idem magnes inermis exerere poteſt. Niſi igitur pondus ferri EF jam tantum fuerit, quantum magnes geſtare valet, magnes inſuper pondus ferramento in C appendendum ſuſtinere poterit. Cum autem jam ferrum EF ipſum magnetis vicem teneat, per vim attractivam onus ferreum adhuc majus ferre poterit, quam ſolus magnes valeret. Hæc autem tam plane ex theoria conſequuntur, ut ſuperfluum ſit pluribus ejusmodi explicationibus immorari, hancque ob rem plurima alia experimenta, quæ magnes naturæ ſcrutatoribus ſuppeditavit, prætermitto; cum & facillime ex theoria deriventur, & ipſa theoria nulla ulteriori confirmatione indigeat. Cum enim Illuſtr. Academia hoc ſolum propoſuiſſet, ut theoria ſeu cauſa phyſica phænomenorum magnetis in lucem protrahatur, quia huic quæſtioni ex aſſe mihi quidem ſatisfeciſſe videor, finem huic tractationi impono.

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NOVA METHODUS
Inveniendi Trajectorias reciprocas Algebraicas.

I.

Tabula. III. Viginti abhinc annis & quod excurrit, hoc problema de trajectoriis reciprocis primum a Nicolao Bernoullio Johannis Filio non ſolum eſt propoſitum; ſed etiam tam variæ & elegantes ſolutiones jam eo tempore ſunt exhibitæ, ut hoc problema jam penitus exhauſtum videri poſſit. Cum enim præcipua difficultas in inveniendis curvis algebraicis huic quæſtioni ſatisfacientibus verſaretur, tradideram equidem in ſecundo Comment. Petropol. Tomo methodum ex quolibet linearum curvarum ordine unam ad minimum inveniendi, quæ præſcripta proprietate ſit affecta. Tanto præterea ſtudio hoc problema illo tempore a pluribus Geometris fuit pertractatum, ut etiamſi ad ſolam Geometriam pertineret, tamen inde univerſa Analyſis tam eximia acceperit augmenta, ut pluribus aliis quæſtionibus majoris momenti enodandis apta ſit reddita, quæ ſine his ſubſidiis intactæ eſſent relictæ.

2. Hanc igitur quæſtionem maxime famoſam denuo aggredior, non quo aliorum ſolutiones minus idoneas vel inſufficientes cenſeam: ſed quoniam tum temporis curvæ algebraicæ, quibus præcipua problematis vis continetur, non levi labore ac per operoſas integrationes ſunt erutæ, neque omnes in formulis generalibus comprehendi potuerunt; explicabo hic methodum ſingularem, cujus ope non

ſolum

ſolum curvæ algebraicæ, quibus problema ſolvitur, facili negotio, & quidem quod maxime paradoxon videatur, ſine ulla integratione inveniri, ſed etiam omnes ſimul finitis formulis contentæ repræſentari queant. Quanquam autem hac methodo jam ſæpius in aliis quæſtionibus ſolvendis ſum uſus, tamen eam nusquam adhuc expoſui; ejusque applicatio ad præſens negotium peculiare requirit artificium, quod in aliis caſibus haud parum utilitatis afferre poterit.

3. Problema autem hoc ſequenti modo proponi eſt ſolitum: Fig. 1.

„*Circa datum axem* ACB *deſcribere ejusmodi lineam* „*curvam* ECF, *quæ circa axem in ſitu inverſo* eCf *conſtituta, at* „*ſecundum directionem axis motu ſibi parallelo promota, in quovis* „*ſitu* c'e'f' *priorem curvam* ECF *ſub dato angulo in* M *interſecet.*

Solutio vero ſequenti modo a Celeb. Joh. Bernoullio ad Analyſin eſt perducta. Cum angulus EMc' ubique debeat eſſe datæ magnitudinis, erit is æqualis angulo ECe, ideoque duplus anguli ECA. Per M ducatur recta MP axi AB parallela; eritque $EMP + c'MP = ECe$; at ob motum parallelum eſt angulus $e'MP = enP$; & ob ſitum inverſum ſi ducatur QN axi AB parallela, ab eoque æquidiſtans ac recta PM, erit ang: $ENQ = enP$. Quare requiritur, ut ductis binis quibusque rectis MP & NQ axi parallelis ab eoque æquidiſtantibus, ſumma angulorum $EMP + ENQ$ ſit ubique eadem atque æqualis duplo anguli ECA.

4. Cum igitur natura quæſtionis ad unam lineam curvam ſit revocata, ducatur ad axem AB recta GH, quæ cum eo faciat angulum CAH æqualem duplo angulo ECA ſeu Fig. 2.

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ſeu ipſi angulo interſectionis propoſito æqualem. In hacque recta capiantur utrinque abſciſſæ AP, AQ, quæ ob æquales applicatarum PM & QN ab axe AB diſtantias erunt æquales; eritque $EMP + ENQ = CAH$. Ductis autem utrinque applicatis infinite propinquis *pm* & *qn*, rectæque GH parallelis $M\mu$ & $n\nu$; ob angulum $M\mu m = N\nu n = GAC$ erit: $Mm\mu + mM\mu = nN\nu + Nn\nu = CAH = EMP + ENQ$. At $Mm\mu = EMP$ & $nN\nu = ENQ$, unde ſequitur fore $mM\mu = nN\nu$ & $Nn\nu = Mm\mu$. Erunt ergo triangula $Mm\mu$ & $Nn\nu$ æquiangula ac propterea ſimilia; ex quo habebitur hæc proportio $m\mu : M\mu = n\nu : N\nu$, ideoque hæc æqualitas $m\mu \cdot N\nu = M\mu \cdot n\nu$, qua natura problematis continetur.

5. Vocemus abſciſſam $AP = x$, eique reſpondentem applicatam $PM = y$; erit abſciſſa ex altera parte ſumta $AQ = -x$, cui reſpondens applicata ponatur $QN = z$; quæ talis erit functio ipſius $-x$, qualis y eſt ipſius $+x$; ſeu ex valore ipſius y prodibit valor ipſius z, ſi loco x ubique ſcribatur $-x$. His poſitis erit $Pp = M\mu = dx$; $\mu m = dy$; $Qq = n\nu = -dx$, & $N\nu = -dz$; atque æquatio modo inventa $m\mu \cdot N\nu = M\mu \cdot n\nu$ dabit hanc formulam

$$-dydz = -dx^2 \text{ ſeu } \frac{dy}{dx} \cdot \frac{dz}{dx} = 1.$$

Ponatur $\frac{dy}{dx} = M$, & $\frac{dz}{dx} = N$, eritque N talis functio ipſius $-x$, qualis M eſt ipſius $+x$, ſeu ex functione M proveniet functio N, ſi loco x ponatur $-x$. Quocirca ad problema reſolvendum ejusmodi functiones pro M inveſtigari oportet, ut fiat $MN = 1$: hocque facto erit $dy = Mdx$, quæ æquatio naturam curvæ exprimet,

6. Quæ-

6. Quæſtio itaque huc eſt perducta, ut pro M ejusmodi inveſtigetur functio ipſius x, quæ, ſi loco x ponatur $-x$, abeat in N, ita ut ſit $MN = 1$. Manifeſtum autem eſt huic conditioni ſatisfacere hujusmodi valores $M = a^x$; $M = a^{x^3}$, ſimilesque alios; ſed cum curvas algebraicas requiramus, hujusmodi valores exponentiales excludi oportet. Sit igitur P functio par ipſius x, quæ ſcilicet eundem valorem retineat, poſito $-x$ loco $+x$; deinde ſit Q functio impar ipſius x, quæ abeat in $-Q$, ſi loco x ponatur $-x$: hincque evidens eſt conditionem problematis impleri, ſi ponatur $M = \frac{P+Q}{P-Q}$, fiet enim $N = \frac{P-Q}{P+Q}$ ideoque $MN = 1$. Ponatur $\frac{Q}{P} = u$, ut ſit u functio quæcunque ipſius x impar; exquo erit $M = \frac{1+u}{1-u}$ & $dy = \frac{1+u}{1-u}\,dx$, quæ æquatio ſolutionem problematis in latiſſimo ſenſu complectitur, dummodo ſub littera u omnes functiones ipſius x comprehendantur.

7. Quamquam hæc æquatio jam eſt generalis omnesque ſolutiones includit, tamen ex ea aliæ formari poſſunt, quæ latius patere videntur; & quæ in inventione curvarum algebraicarum uſum commodiorem præſtant. Hujusmodi eſt iſta formula $M = \left(\frac{1+u}{1-u}\right)^n$; quicunque enim numerus pro exponente n aſſumatur, erit ſemper $N = \left(\frac{1-u}{1+u}\right)^n$,

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ideoque $MN = 1$. Quare ſi u ſumatur pro functione quacunque impari ipſius x, natura curvæ trajectoriæ reciprocæ cujuscunque hac exprimetur æquatione: $dy = \left(\frac{1+u}{1-u}\right)^n dx$. Manifeſtum autem eſt, ſi pro n ſumantur numeri fracti, facile ejusmodi curvas obtineri, quæ ex priori forma difficulter erui queant, etiamſi revera in ea contineantur.

8. Tametſi functiones irrationales ob ambiguitatem neque functionibus paribus neque imparibus proprie annumerari queant: tamen in hoc negotio hujusmodi expreſſiones $\sqrt{(1+uu)}$ pro functionibus paribus haberi poſſunt, dummodo uu ſit functio par neque ex $(1+uu)$ radix quadrata actu extrahi queat. At ſi u ſit functio ipſius x impar, erit uu ac propterea $\sqrt{(1+uu)}$ ejusdem x functio par. Quo notato facile patet, hunc valorem $M = \sqrt{(1+uu)} + u$ quæſito ſatisfacere debere; fiet enim inde $N = \sqrt{(1+uu)} - u$, ideoque $MN = 1$. Idem evenit, ſi ſtatuatur $M = (\sqrt{(1+uu)}+u)^n$, quia fit $N = (\sqrt{(1+uu)}-u)^n$ & $MN = 1$. Hinc ergo duæ novæ æquationes generales pro trajectoriis reciprocis oriuntur:

$$dy = (\sqrt{(1+uu)} + u)\, dx \quad \&$$

$$dy = (\sqrt{(1+uu)} + u)^n\, dx$$

9. Poteſt etiam nova quædam variabilis t introduci, a qua x ita pendeat, ut poſito $-t$ loco t, abſciſſa x abeat in $-x$; ſeu ſit x functio impar ipſius t. Ponatur $dx = vdt$, eritque v functio par ipſius t; ſtatuatur autem ut ante u

functio

functio impar ipſius t. His poſitis, ſi fiat $M = \frac{dy}{dx} = \frac{1+u}{1-u}$, evadet denuo $N = \frac{1-u}{1+u}$, ideoque $MN = 1$. Hancobrem problemati ſatisfiet, ſi ſumatur:

$$dx = vdt \quad \& \quad dy = \frac{1+u}{1-u}\, vdt$$

Simili modo problema ſolvetur his formulis generalibus

$$dx = vdt \quad \& \quad dy = \left(\frac{1+u}{1-u}\right)^n vdt$$

itemque his ex irrationalibus ortis:

$$dx = vdt \;\&\; dy = (\sqrt{(1+uu)}+u)\, vdt \text{ atque}$$

$$dx = vdt \;\&\; dy = (\sqrt{(1+uu)}+u)^n\, vdt.$$

Quæcunque autem formulæ ex his aſſumantur, neceſſe eſt ut inde ſolutio problematis generalis obtineatur.

10. Si jam curvæ algebraicæ deſiderentur, totum negotium huc redit, ut qualitas functionis u, & in his poſterioribus binarum functionum u & v determinetur, qua hæ formulæ integrabiles reddantur. Plures autem imo infinitæ hujusmodi functiones, cum a Celeb. Bernoullio, tum a me ſunt notatæ, quæ curvas algebraicas præbeant; ſed hic modus maxime eſt particularis, neque omnes curvas algebraicas ſatisfacientes in ſe complectitur. Deinde aſſumtis iſtiusmodi functionibus idoneis, integratio harum formularum demum actu inſtitui debet; ſicque pro quovis curvarum genere peculiari operatione eſt opus, quæ ſæpe non ſine moleſto calculo abſolvitur. Cui incommodo ita

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occurram, ut non ſolum formulas generales pro omnibus curvis algebraicis ſim exhibiturus; ſed etiam quæ ſine prævia integratione ſolutionem ſuppeditent. Quin etiam has ipſas formulas algebraicas ex ſuperioribus differentialibus ſine actuali integratione ſum derivaturus, id quod plerisque maxime paradoxon videbitur. Methodum autem meam ad ſingulas formulas differentiales ante inventas ſeorſim accommodabo.

I. Modus inveniendi trajectorias reciprocas algebraicas ex formula

$$dy = \frac{1+u}{1-u}\,dx$$

11. Quæritur ergo hic, non ſolum qualis functio ipſius x debeat eſſe u, ut formula $\frac{1+u}{1-u}\,dx$ integrationem admittat, ſed etiam quænam ipſa ſit futura integralis forma. Cum autem u ſit functio impar ipſius x, erit viciſſim x functio impar ipſius u: hincque inveſtigabo, qualis functio quantitas x eſſe debeat ipſius u, ut quoque y per functionem algebraicam ipſius u exprimi queat. Quam inveſtigationem ita inſtituo: quia eſt $y = \int \frac{1+u}{1-u}\,dx$, erit per notam integralium reductionem:

$$y = \frac{1+u}{1-u}\,x - 2\int \frac{x\,du}{(1-u)^2}.$$

Supereſt ergo, ut formula $\int \frac{x\,du}{(1-u)^2}$ reddatur integrabilis,

lis, in quo nulla foret difficultas, niſi x deberet eſſe functio impar ipſius u.

12. Denotet p functionem quamcunque parem, & q functionem imparem ipſius u, ſtatuaturque:

$$2\int \frac{x\,du}{(1-u)^2} = \frac{(p+q)(1+u)}{1-u}$$

duas ſcilicet novas quantitates p & q introduco, ut non ſolum integrabilitas procuretur, ſed etiam functioni x præſcripta proprietas inducatur. Sumtis ergo differentialibus, erit:

$$\frac{2\,x\,du}{(1-u)^2} = \frac{(dp+dq)(1+u)}{1-u} + \frac{2(p+q)\,du}{(1-u)^2}$$

multiplicationeque per $(1-u)^2$ inſtituta orietur:

$$2\,xdu = (1-uu)\,dp + (1-uu)\,dq + 2\,pdu + 2\,qdu$$

cujus æquationis alii termini pares ipſius u continebunt dimenſiones, alii impares: quamobrem neceſſe eſt ut termini tam parium quam imparium dimenſionum ſeorſim inter ſe æquentur.

13. Quia vero p eſt functio par ipſius u, reliquæ vero quantitates q & x functiones impares, earumque differentialia eandem naturam ſequuntur, erit $2\,xdu$ functio par; $(1-uu)\,dp$ par; $(1-uu)\,dq$ impar; $2\,pdu$ impar; & $2\,qdu$ par. Æquatis ergo paribus & imparibus ſeorſim ſequentes duæ orientur æquationes:

$$2\,xdu = (1-uu)\,dp + 2\,qdu;$$

$$\&\quad 2\,pdu + (1-uu)\,dq = 0.$$

Poſte-

Poſterior æquatio ſtatim definit $p = -\frac{(1-uu)\,dq}{2\,du}$:
hinc enim ob q functionem imparem ipſius u fiet p functio par. Cognito jam valore functionis p, ex priore æquatione deducitur:

$$x = \frac{(1-uu)\,dp}{2\,du} + q:$$ cui abſciſſæ reſpondebit applicata $y = \frac{1+u}{1-u}x - \frac{(p+q)(1+u)}{1-u} = \frac{1+u}{1-u}(x-q-p)$

14. Si hic pro x valor ante inventus ſubſtituatur, invenietur:

$$y = \frac{(1+u)^2 dp}{2\,du} - \frac{p(1+u)}{1-u}$$ ſeu ob $p = -\frac{(1-uu)dq}{2\,du}$

$$y = \frac{(1+u)^2 dp + (1+u)^2 dq}{2\,du} = \frac{(dp+dq)(1+u)^2}{2\,du}$$

Quocirca hinc nanciſcimur

Primam regulam generalem pro inveniendis trajectoriis reciprocis algebraicis:

Sumatur q *functio quæcunque imparium dimenſionum ipſius* u: *indeque quaeratur quantitas* $p = -\frac{(1-uu)\,dq}{2\,du}$, *qua inventa erit curvae quaeſitae:*

abſciſſa $AP = x = q + \frac{(1-uu)\,dp}{2\,du}$

appli-

applicata $PM = y = \frac{(dp+dq)(1+u)^2}{2du} = \frac{1+u}{1-u}(x-p-q)$

qui valores ſemper ſunt algebraici, ſi quidem q *fuerit functio algebraica ipſius* u

15. Si ipſius u alia ſumatur functio impar q', ex eaque capiatur $p' = -\frac{(1-uu)dq'}{2du}$, habebitur ſimili modo:

$$x = q' + \frac{(1-uu)dp'}{2du}. \; \& \; y = \frac{(dp'+dq')(1+u)^2}{2du}$$

Atque cum hinc æque fiat $\frac{dy}{dx} = \frac{1+u}{1-u}$: manifeſtum eſt ſi ex quapiam ipſius q hypotheſi inventum fuerit:

$$x = X \;\&\; y = Y$$

ex alia autem hypotheſi prodierit $x = X'$ & $y = Y'$, problemati quoque ſatisfieri his valoribus:

$$x = X + X' \;\&\; y = Y + Y'$$

atque generalius etiam, ſi ponatur:

$$x = \alpha X + \beta X' \;\&\; y = \alpha Y + \beta X'$$

ſicque ex duabus curvis algebraicis jam inventis innumerabiles novæ inveniri poterunt. Sin autem præterea tertia fuerit inventa $x = X''$ & $y = Y''$, erit quoque $x = \alpha X + \beta X' + \gamma X''$ & $y = \alpha Y + \beta Y' + \gamma X''$ ſicque porro.

16. Ponatur ut ad exempla deſcendamus $q = u^{\lambda}$ exiſtente λ numero quocunque impari, ſive integro ſive fracto, cujus tam numerator quam denominator ſit impar: eritque

$\frac{dq}{du} = \lambda u^{\lambda-1}$ & $p = -\frac{1}{2}\lambda u^{\lambda-1} + \frac{1}{2}\lambda u^{\lambda+1}$ hincque porro

dp

$\frac{dp}{du} = -\frac{1}{2}\lambda(\lambda-1)u^{\lambda-2} + \frac{1}{2}\lambda(\lambda+1)u^{\lambda}$. Unde habebitur:

$$x = u^{\lambda} - \frac{1}{4}\lambda(\lambda-1)u^{\lambda-2} + \frac{1}{4}\lambda(\lambda+1)u^{\lambda} + \frac{1}{4}\lambda(\lambda-1)u^{\lambda}$$
$$- \frac{1}{4}\lambda(\lambda+1)u^{\lambda+2}$$

ſeu $x = -\frac{1}{4}\lambda(\lambda-1)u^{\lambda-2} + \frac{1}{2}(\lambda\lambda+2)u^{\lambda} - \frac{1}{4}\lambda(\lambda+1)u^{\lambda+2}$

& $y = \frac{1}{2}\lambda(u^{\lambda-1} - \frac{1}{2}(\lambda - \ )u^{\lambda-2} + \frac{1}{2}(\lambda+1)u^{\lambda})(1+u)^2$ ſeu

$$y = \frac{1}{4}\lambda u^{\lambda-2}(1+u)^4 - \frac{1}{4}\lambda\lambda u^{\lambda-2}(1-u)(1+u)^3 \text{ vel etiam}$$

$$y = \frac{1}{4}\lambda u^{\lambda-2}(1-\lambda+u+\lambda u)(1+u)^3$$

Hinc ſequitur fore:

ſi $\lambda = 1$: $x = \frac{3}{2}u - \frac{1}{2}u^3$; $y = \frac{1}{2}(1+u)^3$ pro parabola cubicali ſecunda

ſi $\lambda = 3$; $x = -\frac{1}{2}u(1-3uu)(3-2uu)$: &

$$y = -\frac{3}{2}u(1-2u)(1+u)^3$$

ſi $\lambda = 5$; $x = -\frac{1}{2}u^3(10-27u^2+15u^4)$: &

$$y = -\frac{5}{2}u^3(2-3u)(1+u)^3$$

ſi $\lambda = 7$; $x = -\frac{1}{2}u^5(21-51u^2+28u^4)$: &

$$y = -\frac{7}{2}u^5(3-4u)(1+u)^3$$

&c.

II. Se-

II. Secundus Modus Inveniendi Trajectorias reciprocas Algebraicas.

Ex formula $dy = \left(\frac{1+u}{1-u}\right)^n dx$

17. Quia ut ante oſtenſum eſt, abſciſſa x debet eſſe functio impar ipſius u, quæratur qualis eſſe debeat, ut applicata y prodeat algebraice expreſſa. In hunc finem ponatur:

$$y = \int \left(\frac{1+u}{1-u}\right)^n dx = \left(\frac{1+u}{1-u}\right)^n x - 2n \int \frac{x\,du}{1-uu} \left(\frac{1+u}{1-u}\right)^n$$

Sumtisque p functione pari & q functione impari ipſius u ſtatuatur:

$$\int \left(\frac{1+u}{1-u}\right)^n \frac{x\,du}{1-uu} = (p+q)\left(\frac{1+u}{1-u}\right)^n$$

ex qua differentiatione inſtituta habebitur;

$$\frac{x\,du}{1-uu} = dp + dq + \frac{2n(p+q)\,du}{1-uu} \quad \text{ſeu}$$

$$x\,du = (1-uu)\,dp + (1-uu)\,dq + 2npdu + 2nqdu$$

18. Diſcerpatur hæc in duas æquationes, quarum altera contineat functiones parium, altera vero imparium dimenſionum, ſicque fiet:

$$x\,du = (1-uu)\,dp + 2nqdu$$

$$\& \; 0 = (1-uu)\,dq + 2npdu$$

 qua-

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quarum poſterior dat $p = -\frac{(1 - uu)\,dq}{2ndu}$, quo valore ipſius p invento erit ex priori:

$$x = 2nq + \frac{(1 - uu)\,dp}{du}$$

Præterea autem ex ſupra facta hypotheſi orietur

$$y = \left(\frac{1+u}{1-u}\right)^{n} (x - 2np - 2nq) = \left(\frac{1+u}{1-u}\right)^{n} \frac{(1-uu)(dp+dq)}{du}$$

ſeu $y = \frac{(dp+dq)(1+u)^{n+1}}{du(1-u)^{n-1}}$. Sumta ergo pro q functione quacunque imparium dimenſionum ipſius u, hæ formulæ præbebunt curvas algebraicas, quæ erunt trajectoriæ reciprocæ.

19. Hinc ergo adepti ſumus:

Secundam regulam generalem pro inveniendis trajectoriis reciprocis algebraicis.

Sumatur q *functio quæcunque imparium dimenſionum ipſius* u, *denotanteque* n *numerum quemcunque, quæratur inde quantitas* $p = -\frac{(1 - uu)\,dq}{2ndu}$; *hinc erit curvae quaeſitae;*

Abſciſſa $AP = x = 2uq + \frac{(1 - uu)\,dp}{du}$

Appli-

Applicata PM $= y = \frac{(dp+dq)(1+u)^{n+1}}{du(1-u)^{n-1}}$

Ubi iterum notandum eſt, ſi pro x & y jam aliquot inventi fuerint valores X & Y, X' & Y', X'' & Y'', &c. problemati quoque ſatisfieri, ſi capiatur

$$x = \alpha X + \beta X' + \gamma X'' \text{ \&c. } \quad \& \; y = \alpha Y + \beta Y' + \gamma Y'' \text{ \&c.}$$

20. Sit λ numerus quicunque impar ac ſtatuatur: $q = 2nu^{\lambda}$, erit $\frac{dq}{du} = 2n\lambda u^{\lambda-1}$ ideoque $p = -\lambda u^{\lambda-1} + \lambda u^{\lambda+1}$; unde fit porro $\frac{dp}{du} = -\lambda(\lambda-1)u^{\lambda-2} + \lambda(\lambda+1)u^{\lambda}$. Ex his valoribus colligitur:

$$x = 4nn\,u^{\lambda} - \lambda(\lambda-1)u^{\lambda-2} + 2\lambda\lambda u^{\lambda} - \lambda(\lambda+1)u^{\lambda+2} \;\&$$

$$y = -(\lambda(\lambda-1)u^{\lambda-2} - 2n\lambda u^{\lambda-1} - \lambda(\lambda+1)u^{\lambda})\,\frac{(1+u)^{n+1}}{(1-u)^{n-1}}$$

ſeu mutatis ſignis, ſumtisque ſemiſſibus erit

$$x = u^{\lambda-2}\left(\frac{\lambda(\lambda-1)}{2} - (\lambda\lambda+2nn)u^{2} + \frac{\lambda(\lambda+1)}{2}u^{4}\right)$$

$$y = \lambda u^{\lambda-2}\left(\frac{\lambda-1}{2} - nu - \frac{(\lambda+1)uu}{2}\right)\frac{(1+u)^{n+1}}{(1-u)^{n-1}}$$

Hinc ergo erit:

$$\text{ſi } \lambda = 1:\; x = u^3 - (1+2nn)u;\; y = -(n+u)\frac{(1+u)^{n+1}}{(1-u)^{n-1}}$$

$$\text{ſi } \lambda = 3:\; x = u(3-(9+2nn)u^2+6u^4);\; \&$$

$$y = 3u(1-nu-2uu)\,\frac{(1+u)^{n+1}}{(1-u)^{n-1}}$$

$$\text{ſi } \lambda = 5;\; x = u^3(10-(15+2nn)u^2+15u^4);\; \&$$

$$y = 5u^3(2-nu-3uu)\,\frac{(1+u)^{n+1}}{(1-u)^{n-1}}$$

&c.

21. Sit porro m numerus quicunque, ac ponatur $q = 2nu^{\lambda}(1-uu)^m$, erit $\frac{dq}{du} = 2n\lambda u^{\lambda-1}(1-uu)^m$ $-4mnu^{\lambda+1}(1-uu)^{m-1}$ ac propterea $p = -\lambda u^{\lambda-1}(1-uu)^{m+1} + 2mu^{\lambda+1}(1-uu)^m$. Fietque

$$\frac{dp}{du} = -\lambda(\lambda-1)u^{\lambda-2}(1-uu)^{m+1} + 2\lambda(m+1)u^{\lambda}(1-uu)^m$$
$$+2m(\lambda+1)u^{\lambda}(1-uu)^m$$
$$-4mmu^{\lambda+2}(1-uu)^{m-1}$$

ex quibus colligitur:

$x =$

$$x = 4nnu^{\lambda}(1-uu)^{m} - \lambda(\lambda-1)u^{\lambda-2}(1-uu)^{m+2}$$
$$+2(2m\lambda+m+\lambda)u^{\lambda}(1-uu)^{m+1}$$
$$-4mmu^{\lambda+2}(1-uu)^{m}$$

ſeu

$$x = u^{\lambda-2}(1-uu)^{m}(-\lambda(\lambda-1)+2(2nn+\lambda\lambda+2m\lambda+m)u^{2}$$
$$-(4mm+4m\lambda+\lambda\lambda+2m+\lambda)u^{4})$$

ſive hoc modo brevius:

$$x = -u^{\lambda-2}(1-uu)^{m}(\lambda(\lambda-1)-2(2nn+\lambda\lambda+2m\lambda+m)u^{2}$$
$$+(2m+\lambda)(2m+\lambda+1)u^{4})$$

Deinde erit:

$$y = \frac{(1+u)^{n+1}}{(1-u)^{n-1}}(2n\lambda u(1-uu)-4mnu^{3}-\lambda(\lambda-1)(1-uu)^{2}$$
$$-4mmu^{4}+2(2m\lambda+m+\lambda)u^{2}(1-uu))u^{\lambda-2}(1-uu)^{m-1}$$

$$\text{ſeu } y = -\left(\frac{1+u}{1-u}\right)^{n}u^{\lambda-2}(1-uu)^{m}(\lambda(\lambda-1)-2(\lambda\lambda+$$
$$2m\lambda+m)u^{2}+(2m+\lambda)(2m+\lambda+1)u^{4}$$
$$-2n\lambda u+2n(2m+\lambda)u^{3})$$

mutatis ergo ſignis erit:

$$x = u^{\lambda-2}(1-uu)^m(\lambda(\lambda-1) - 2(2nu+\lambda\lambda+2m\lambda+m)u^2 + (2m+\lambda)(2m+\lambda+1)u^4)$$

$$y = u^{\lambda-2}(1-uu)^m\left.\begin{array}{l}(\lambda(\lambda-1) - 2(\lambda\lambda+2m\lambda+m)u^2 \\ +(2m+\lambda)(2m+\lambda+1)u^4 \\ -2n\lambda u + 2n(2m+\lambda)u^3\end{array}\right\}\left(\frac{1+u}{1-u}\right)^n$$

22. Hinc duæ prodeunt ſolutiones præ ceteris ſimpliciores, quarum altera prodit, ſi $\lambda = 1$ & $m = -1$: tum enim fiet:

$$x = -\frac{4(nn-1)u}{1-uu} \;\&\; y = -\frac{2(n-2u+nuu)}{1-uu}\left(\frac{1+u}{1-u}\right)^n$$

ſeu ſi utrinque per conſtantem $\frac{-a}{2(nn-1)}$ multiplicetur erit

$$x = \frac{2au}{1-uu} \;\&\; y = \frac{a(n-2u+nuu)}{(nn-1)(2-uu)}\left(\frac{1+u}{1-u}\right)^n$$

unde non difficulter eliminatur variabilis u; eſt enim ex priori $u = \frac{-a+\sqrt{(aa+xx)}}{x}$: quo valore in altera ſubſtituto, irrationalibusque ex denominatore ſublatis pervenietur tandem ad hanc æquationem:

$$(nn-1)a^n y = (n\sqrt{(aa+xx)}-x)(\sqrt{(aa+xx)}+x)^n$$

Ad

Ad quam reducendam notetur eſſe: $(\sqrt{(aa+xx)}+x)^n =$

$$a^n + \frac{nn}{1.2} a^{n-4} xx + \frac{nn(nn-4)}{1.2.3.4} a^{n-4} x^4 + \frac{nn(nn-4)(nn-16)}{1.2.3.4.5.6} a^{n-6} x^6 + \&c.$$

$$+ \left(na^{n-2} x + \frac{n(nn-4)}{1.2.3.} a^{n-4} x^3 + \frac{n(nn-4)(nn-16)}{1.2.3.4.5} a^{n-6} x^5 + \&c.\right) \sqrt{(aa+xx)}$$

unde reperietur fore:

$$(nn-1)a^n y = \left\{ \begin{aligned} &\left(na^n + \frac{n(nn-2)}{1.2} a^{n-2} x^2 + \frac{n(nn-4)(nn-4)}{1.2.3.4} a^{n-4} x^4 + \frac{n(nn-6)(nn-4)(nn-16)}{1.2.3.4.5.6} a^{n6} x^6 + \&c.\right)\sqrt{(aa+xx)} \\ &+ (nn-1)a^n x + \frac{nn(nn-1)}{1.2.3} a^{n-2} x^3 + \frac{nn(nn-1)(nn-4)}{1.2.3.4.5} a^{n-4} x^5 + \frac{nn(nn-1)(nn-4)(nn-16)}{1.2.3.4.5.6.7} a^{n-6} x^7 \ \&c. \end{aligned} \right.$$

quæ abrumpitur, quoties eſt n numerus par. Ponatur $a = 1$ ſitque exempli gr.

$n = 1$ erit $y = x$

$n = 2$; $3y = (2+2xx)\sqrt{(1+xx)} + 3x + 2x^3$

$n =$

$$n = 4;\ 15y = (4 + 28xx + 24x^4)\sqrt{(1+xx)} + 15x + 40x^3 + 24x^5$$

$$n = 6;\ 35y = (6 + 102xx + 256x^4 + 160x^6)\sqrt{(1+xx)} + 35x + 210x^3 + 336x^5 + 160x^7$$

&c.

23. Sin autem n ſit numerus impar, erit

$$(\sqrt{(aa+xx)} + x)^n = na^{n-1}x + \frac{n(nn-1)}{1.2.3}a^{n-3}x^3 + \frac{n(nn-1)(nn-9)}{1.2.3.4.5}a^{n-5}x^5 + \&c. +$$

$$(a^{n-1} + \frac{(nn-1)}{1.2.}a^{n-3}x^2 + \frac{(nn-1)(nn-9)}{1.2.3.4}a^{n-5}x^4 + \frac{(nn-1)(nn-9)(nn-25)}{1.2.3.4.5.6}a^{n-7}x^6 + \&c.)\sqrt{(aa+xx)}$$

unde ſequitur fore:

$$(nn-1)a^n y =$$

$$na^{n+1} + \frac{n(nn-1)}{1.2}a^{n-1}x^2 + \frac{n(nn-1)(nn-1)}{1.2.3.4}a^{n-3}x^4 + \frac{n(nn-1)(nn-1)(nn-9)}{1.2.3.4.5.6}a^{n-5}x^6 + \&c.$$

$$+ ((nn-1)a^{n-1}x + \frac{(nn-3)(nn-1)}{1.2.3}a^{n-3}x^3 + \frac{(nn-5)(nn-1)(nn-9)}{1.2.3.4.5}a^{n-5}x^5 + \&c.)\sqrt{(aa+xx)}$$

fit

Sit $a = 1$, ac ponatur ſucceſſive $n = 1, 3, 5$, &c.

$n = 1;\quad 0y = 1$

$n = 3;\quad 8y = 3 + 12xx + 8x^4 + (8x + 8x^3)\sqrt{(1+xx)}$

$n = 5;\quad 24y = 5 + 60x^2 + 120x^4 + 64x^6 + (24x + 88x^3 + 64x^5)\sqrt{(1+xx)}$

Generaliter autem poſito $a = 1$ erit: $(nn-1)y =$

$$(n-1)2^{n-1}x^{n+1} + \frac{(n+1)n}{1}2^{n-3}x^{n-1} + \frac{(n+1)n(n-1)}{1.2}2^{n-5}x^{n-3} + \frac{(n+1)n(n-1)(n-4)}{1.2.3}2^{n-7}x^{n-5} +$$

$$\frac{n(nn-1)(n-5)(n-6)}{1.2.3.4}2^{n-9}x^{n-7} + \frac{n(nn-1)(n-6)(n-7)(n-8)}{1.2.3.4.5}2^{n-11}x^{n-9} + \&c. +$$

$$((n-1)2^{n-1}x^{n} + \frac{nn-n+2}{1}2^{n-3}x^{n-2} + \frac{(nn-n+4)(n-3)}{1.2}2^{n-5}x^{n-4} + \frac{(nn-n+6)(n-4)(n-5)}{1.2.3}2^{n-7}x^{n-6}$$

$$+ \frac{(nn-n+8)(n-5)(n-6)(n-7)}{1.2.3.4}2^{n-9}x^{n-8} + \&c.)\sqrt{(1+xx)}$$

24. Si n fuerit numerus fractus puta $= \frac{n}{m}$, habebitur pro trajectoria reciproca hæc æquatio:

$$(nn-mm)y = m(n\sqrt{(1+xx)}-mx)(\sqrt{(1+xx)}+x)^{n:m}$$

ſeu

$$\left(\frac{nn-mm}{m}\right)^m y^m = (n\sqrt{(1+xx)}-mx)^m (\sqrt{(1+xx)}+x)^n$$

quæ maxime fœcunda eſt in curvis ſimplicioribus ſuppeditandis: exceptis enim ordinibus 2 & 3, ex quovis ordine adminimum unam largitur trajectoriam reciprocam. Convenit autem hæc æquatio cum ea, quam in Tomo Comment. II. exhibueram. Scilicet ſi $m = 1$, curva erit ordinis $n + 2$: æquatio enim ab irrationalitate liberata non plures dimenſiones obtinet quam $n + 2$: ſin autem denominator m fuerit numerus quicunque, æquatio ad rationalitatem reducta aſſurget ad $n + 2m$ dimenſiones. Deſideratur autem adhuc methodus has æquationes rationales ſine operoſa elevatione ad poteſtates ex traditis formulis ſtatim eliciendi; cujusmodi methodus ſine dubio in Analyſi datur, cum in ſublatione ſurditatis plurimi termini ſe mutuo deſtruant. Æquatio vero rationalis ita erit comparata, ut ſit

$y^{2m} + Py^m + Q = 0$, ubi ſint P & Q functiones rationales ipſius x.

25. Per inductionem autem ex pluribus caſibus collegi, ſi ſit $m = 1$, fore

(nn-

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$$(nn-1)y^2 - 2(nn-1)y\left\{\begin{array}{l}(n-1)2^{n-1}x^{n+1} + \frac{(n+1)n}{1}2^{n-3}x^{n-1} \\ + \frac{n(nn-1)}{1.2}2^{n-5}x^{n-3} + \\ \frac{n(nn-1)(n-4)}{1.\ 2.\ 3}2^{n-7}x^{n-5} + \\ \frac{n(nn-1)(n-5)(n-6)}{1.\ 2.\ 3.\ 4}2^{n-9}x^{n-7} + \\ \&c.\end{array}\right\}$$

$$= \pm nn \pm (nn-1)xx$$

ubi ſignorum ambiguorum valet ſuperius, ſi n ſit numerus par, inferius ſi ſit impar. Hinc ergo ſequentes orientur æquationes ſimpliciores pro trajectoriis reciprocis;

$$9y^2 = 6y(2x^3 + 3x) + 3xx + 4$$

$$64y^2 = 16y(8x^4 + 12xx + 3) - 8xx - 9$$

$$225y^2 = 30y(24x^5 + 40x^3 + 15x) + 15xx + 16$$

$$576y^2 = 48y(64x^6 + 120x^4 + 60x^2 + 5) - 24xx - 25$$

$$1225y^2 = 70y(16x^7 + 336x^5 + 210x^3 + 35x) + 35xx + 36$$

$$2304y^2 = 96y(384x^8 + 896x^6 + 672x^4 + 168x^2 + 7) - 48xx - 49$$

$$3969y^2 = 126y(896x^9 + 2304x^7 + 2016x^5 + 672x^3 + 63x) + 63xx + 64$$

quæ

quæ ergo æquationum ſeries, quousque libuerit facile continuabitur.

26. Supra duarum ſolutionum ſimpliciorum, quæ ex formulis (21) ſequuntur, mentionem fecimus, alteramque hic fuſius ſumus proſecuti. Altera igitur ſolutio expendenda eſt, in qua $\lambda = 1$ & $m = -\frac{1}{2}$. Hinc ergo oritur:

$$x = \frac{1}{u\sqrt{(1-uu)}}\left(-(4nn-1)uu\right) = -\frac{(4nn-1)u}{\sqrt{(1-uu)}}$$

$$y = \frac{1}{u\sqrt{(1-uu)}}(uu-2nu)\left(\frac{1+u}{1-u}\right)^n = \frac{u-2n}{\sqrt{(1-uu)}}\left(\frac{1+u}{1-u}\right)^n.$$

Multiplicentur hæ formulæ per $\frac{-a}{4nn-1}$, habebiturque

$$x = \frac{au}{\sqrt{(1-uu)}}; \ \& \ y = \frac{a(2n-u)}{(4nn-1)\sqrt{(1-uu)}}\left(\frac{1+u}{1-u}\right)^n$$

unde fit $\frac{(4nn-1)y}{x} = \frac{2n-u}{u}\left(\frac{1+u}{1-u}\right)^n$. Ex priore autem valore oritur $u = \frac{x}{\sqrt{(aa+xx)}}$, qui in hac ſubſtitutus dat:

$$(4nn-1)y = (2n\sqrt{(aa+xx)}-x)\left(\frac{\sqrt{(aa+xx)}+x}{\sqrt{(aa+xx)}-x}\right)^n$$

ſive:

$$(4nn-1)a^{2n}y = (2n\sqrt{(aa+xx)}-x)(\sqrt{(aa+xx)}+x)^{2n},$$

quæ æquatio non differt ab ea, quam § 22. nacti ſumus, niſi quod hic ſit $2n$, quod ibi erat n

III.

III. Tertius Modus Inveniendi Trajectorias reciprocas Algebraicas ex formula

$$dy = (\sqrt{(1+uu)}+u)^n dx$$

27. Ob d. $(\sqrt{(1+uu)}+u)^n = \frac{ndu}{\sqrt{(1+uu)}}(\sqrt{(1+uu)}+u)^n$ erit:

$$y = x(\sqrt{(1+uu)}+u)^n - n\int\frac{xdu}{\sqrt{(1+uu)}}(\sqrt{(1+uu)}+u)^n$$

ubi notandum eſt, ut ante, eſſe oportere x functionem imparem ipſius u. Sit p functio quæcunque par ipſius u & q functio impar, ſtatuaturque

$$\int\frac{xdu}{\sqrt{(1+uu)}}(\sqrt{(1+uu)}+u)^n = (p+q)(\sqrt{(1+uu)}+u)^n$$

erit ſumtis differentialibus, diviſioneque per $(\sqrt{(1+uu)}+u)^n$ peracta.

$$\frac{xdu}{\sqrt{(1+uu)}} = dp + dq + \frac{npdu+nqdu}{\sqrt{(1+uu)}}$$

Jam hinc formentur duæ æquationes, quarum altera functiones pares, altera impares complectatur:

$$\frac{xdu}{\sqrt{(1+uu)}} = dp + \frac{nqdu}{\sqrt{(1+uu)}} \;\&\; dq + \frac{npdu}{\sqrt{(1+uu)}} = 0$$

quarum poſterior ſponte dat: $p = -\frac{dq\sqrt{(1+uu)}}{ndu}$

Inventa autem functione p æquatio prior præbet:

$$x = nq + \frac{dp\sqrt{(1+uu)}}{du}$$ & ex ſubſtitutione habebimus

$$y = (x - np - nq)(\sqrt{(1+uu)}+u)^n = \frac{(dp+dq)\sqrt{(1+uu)}}{du}(\sqrt{(1+uu)}+u)^n$$

28. Hinc itaque adipiſcimur

Tertiam regulam pro inveniendis trajectoriis reciprocis algebraicis.

Sumatur q *functio quæcunque imparium dimenſionum ipſius* u, *indeque formetur functio* $p = -\frac{dq\sqrt{(1+uu)}}{ndu}$, *qua inventa erit curvæ quaeſitae*

Abſciſſa $AP = x = nq + \frac{dp\sqrt{(1+uu)}}{du}$

Applicata $PM = y = \frac{(dp+dq)(1+uu)}{du}(\sqrt{(1+uu)}+u)^n$

Vel cum ſumto elemento du conſtante ſit $dp = -\frac{ddq\sqrt{(1+uu)}}{ndu} - \frac{udq}{n\sqrt{(1+uu)}}$, erit curvæ quæſitæ:

Abſciſſa $AP = x = nq - \frac{udq}{ndu} - \frac{ddq(1+uu)}{ndu^2}$

Applicata $PM = y = \left(\frac{dq\sqrt{(1+uu)}}{du} - \frac{udq}{ndu} - \frac{ddq(1+uu)}{ndu^2}\right)(\sqrt{(1+uu)}+u)^n$

Ubi

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Ubi iterum notandum eſt, ſi ex aliquot hypotheſibus pro q factis, pro x & y jam inventi fuerint valores X & Y; X′ & Y′; X″ & Y″, &c. quæſtioni quoque ſatisfieri his valoribus

$$x = \alpha X + \beta X' + \gamma X'' + \delta X''' \ \&c.$$

$$y = \alpha Y + \beta Y' + \gamma Y'' + \delta Y''' \ \&c.$$

quo pacto numerus curvarum inventarum facile in infinitum augetur.

29. Ponamus pro q poteſtatem quamcunque ipſius u, cujus exponens λ ſit numerus impar: ſit ſcilicet $q = u^{\lambda}$ erit $\frac{dq}{du} = \lambda u^{\lambda-1}$ & $\frac{ddq}{du^2} = \lambda(\lambda - 1)u^{\lambda-2}$; hinc ergo fiet

$$x = nu^{\lambda} - \frac{\lambda u^{\lambda}}{n} - \frac{\lambda(\lambda-1)}{n}(u^{\lambda-2} + u^{\lambda}) = \frac{(nn - \lambda\lambda)}{n}u^{\lambda} - \frac{\lambda(\lambda-1)}{n}u^{\lambda-2}$$

$$y = \frac{\left(\lambda u^{\lambda-1}\sqrt{(1+uu)} - \frac{\lambda\lambda}{n}u^{\lambda} - \frac{\lambda(\lambda-1)}{n}u^{\lambda-2}\right)}{(\sqrt{(1+uu)}+u)^{n}}$$

$$\text{seu } y = \frac{-\frac{\lambda u^{\lambda-2}}{n}\left(\lambda - 1 - nu\sqrt{(1+uu)} + \lambda u^{2}\right)}{(\sqrt{(1+uu)}+u)^{n}}$$

Unde exponenti λ variis valoribus tribuendis reperietur, poſtquam hi valores per n fuerint multiplicati.

$\lambda =$

$$\lambda = 1;\ \begin{cases} x = (nn-1)\,u \\ y = (n\sqrt{(1+uu)}-u)(\sqrt{(1+uu)}+u)^n \end{cases}$$

$$\lambda = 3;\ \begin{cases} x = (nn-9)\,u^3 - 6u \\ y = 3u(nu\sqrt{(1+uu)}-2-3uu)(\sqrt{(1+uu)}+u)^n \end{cases}$$

$$\lambda = 5;\ \begin{cases} x = (nn-25)\,u^5 - 20u^3 \\ y = 5u^3(nu\sqrt{(1+uu)}-4-5uu)(\sqrt{(1+uu)}+u)^n \end{cases}$$

$$\lambda = 7;\ \begin{cases} x = (nn-49)\,u^7 - 42u^5 \\ y = 7u^5(nu\sqrt{(1+uu)}-6-7uu)(\sqrt{(1+uu)}+u)^n \end{cases}$$

&c.

30. Dividantur formulæ primi caſus per $nn-1$, atque prodibunt iſti valores:

$$x = u \quad \& \ (nn-1)\,y = (n\sqrt{(1+uu)}-u)(\sqrt{(1+uu)}+u)^n$$

unde ob $u = x$ variabilis u facillime eliminatur; orieturque ſequens æquatio inter x & y

$$(nn-1)\,y = (n\sqrt{(1+xx)}-x)(\sqrt{(1+xx)}+x)^n$$

quæ eſt eadem æquatio, quam jam ſupra bis elicuimus, & quam maxime fœcundam eſſe linearum algebraicarum ſimpliciorum, fuſius oſtendi. Hæc enim æquatio ad rationalitatem reducta, ſi fuerit n numerus integer, aſcendet ad $n+2$ dimenſiones, ſin autem n ſit numerus fractus, puta $= \frac{n}{m}$ numerus dimenſionum erit $= n + 2m$. Cum autem poſi-

poſitio $n = 1$ ſit inanis, curva ſimpliciſſima hinc oriunda erit linea quarti ordinis.

31. Jam conjunctim ſumantur formulæ caſuum: $\lambda = 1$ & $\lambda = 3$, ex iisque reperietur:

$$x = (nn-1)\alpha u + (nn-9)\beta u^3 - 6\beta u$$

$$y = \left\{ \begin{matrix} \alpha(n\sqrt{(1+uu)} - n \\ 2\beta u(nu\sqrt{(1+uu)} - 2 - 3uu) \end{matrix} \right\} (\sqrt{(1+uu)}+u)^n$$

Quo hinc facilius variabilis u eliminari poſſit, ponatur $\beta = \frac{1}{nn-9}$ & $(nn-1)\alpha = 6\beta = \frac{6}{nn-9}$; ut ſit $\alpha = \frac{6}{(nn-1)(nn-9)}$, & $x = u^3$, ideoque $u = \sqrt[3]{x}$.

Quare ob $(\sqrt{(1+uu)}+u)^n = (\sqrt{(1+uu)}-u)^{-n}$ erit:

$$y(\sqrt{(1+uu)}-u)^n = \frac{6n\sqrt{(1+uu)} - 6u}{(nn-1)(nn-9)} +$$

$$\frac{3nuu\sqrt{(1+uu)} - 6u - 9u^3}{nn-9} = \frac{3n(2+(nn-1)uu)\sqrt{(1+uu)}}{(nn-1)(nn-9)}$$

$$- \frac{3u(2nn+3(nn-1)uu)}{(nn-1)(nn-9)}.$$ Hancobrem habebimus:

$$\frac{(nn-1)(nn-9)}{3} y(\sqrt{(1+uu)}-u)^n = \begin{matrix} +n(2+(nn-1)uu)\sqrt{(1+uu)} \\ -u(2nn+3(nn-1)uu) \end{matrix}$$

exiſtente $u = \sqrt[3]{x}$.

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32. Conjungamus cum his formulas caſus $\lambda = 5$ eritque

$$x = \alpha u^3 + (nn - 25)\beta u^5 - 20\beta u^3$$

$$\frac{(nn-1)(nn-9)}{3}\, y\,(\surd(1+uu)-u)^n =$$

$$+ n\alpha(2+(nn-1)uu)\surd(1+uu)$$

$$- \alpha u(2nn + 3(nn-1)uu)$$

$$+ \tfrac{5}{3}\beta(nn-1)(nn-9)u^3(nu\surd(1+uu) - 4 - 5uu)$$

Sit $\beta = \frac{1}{nn-25}$ & $\alpha = 20\beta = \frac{20}{nn-25}$, erit $x = u^5$ &

$$\frac{(nn-1)(nn-9)(nn-25)}{5}\, y(\surd(1+uu)-u)^n =$$

$$+ n(24 + 12(nn-1)uu + (nn-1)(nn-9)u^4)\surd(1+uu)$$

$$- u(24nn + 4nn(nn-1)uu + 5(nn-1)(nn-9)u^4)$$

Simili autem modo ulterius pergere licet, his formulis cum caſu $\lambda = 7$ conjungendis: prodibit autem

$x = u^7$ atque

$$\frac{(nn-1)(nn-9)(nn-25)(nn-49)}{7}\, y(\surd(1+uu)-u)^n =$$

$$n(720 + 360(nn-1)u^2 + 30(nn-1)(nn-9)u^4 + (nn-1)(nn-9)(nn-25)u^6)\surd(1+uu)$$

$-u$

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$$-u(720nn+120nn(nn-1)u^2+6nn(nn-1)(nn-9)u^4+$$
$$7(nn-1)(nn-9)(nn-25)u^6)$$

unde non difficulter lex ſequentium formularum colligi poteſt.

33. Præter hos vero caſus, quibus variabilis u facile eliminatur, ex formulis ante §. 29 inventis, alii eadem prærogativa gaudentes derivari poſſunt. Cum enim ſit:

$$x=-(mm-\lambda\lambda)u^{\lambda}+\lambda(\lambda-1)u^{\lambda-2}$$
$$y=\lambda u^{\lambda-2}(\lambda-1-nu\sqrt{(1+uu)}+\lambda uu)(\sqrt{(1+uu)}+u)^n$$

quoties exponens n eſt numerus impar ſive integer ſive fractus, ſtatuere licebit $\lambda=n$. Sit igitur n numerus impar, ac fiat $\lambda=n$ erit:

$$x=n(n-1)u^{n-2} \quad \&$$
$$y=nu^{n-2}(n-1-nu\sqrt{(1+uu)}+nuu)(\sqrt{(1+uu)}+u)^n$$

ſeu per $n(n-1)$, dividendo habebitur:

$$x=u^{n-2} \quad \&$$
$$(n-1)y=u^{n-2}(n-1-nu\sqrt{(1+uu)}+nuu)(\sqrt{(1+uu)}+u)^n$$

34. Si $n=1$ ex his formulis nihil oritur; fiat ergo, ut exemplum afferamus, $n=3$, eritque $x=u$ &

$$y=3x(2-3x\sqrt{(1+xx)}+3xx)(\sqrt{(1+xx)}+x)^3.$$

 Cum

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Cum igitur ſit:

$$(\sqrt{(1+xx)}+x)^3 = 3x+4x^3+(1+4xx)\sqrt{(1+xx)}, \text{ erit}$$

$$y = 3x(3x+2x^3+2(1+xx)\sqrt{(1+xx)}) \quad \text{ideoque}$$

$$y-9xx-6x^4 = 6x(1+xx)\sqrt{(1+xx)}$$

Quæ æquatio ad rationalitatem perducta fit:

$$yy-18xxy-12x^4y = 36xx+27x^4$$

quæ eſt nova linea quinti ordinis in numerum trajectoriarum reciprocarum referenda. Innumerabiles autem aliæ æquationes rationales inter x & y hinc erui poſſunt; ponendis pro n aliis numeris imparibus, quæ autem ad multo altiores dimenſiones aſſurgent.

35. Simpliciores provenient, ſi in prima æquatione ponatur $\lambda = n-2$, unde fit:

$$x = (n-2)(n-3)u^{n-4} - 4(n-1)u^{n-2} \quad \&$$

$$y = (n-2)u^{n-4}(n-3+(n-2)uu-(n-2)u\sqrt{(1+uu)})(\sqrt{(1+uu)}+u)^n$$

Si enim hæ formulæ cum prius inventis conjungantur, fiatque

$$x = \alpha u^{n-2} + (n-2)(n-3)\beta u^{n-4} - 4(n-1)\beta u^{n-2}$$

$$\left.\begin{array}{l}(n-1)y = \alpha u^{n-2}(n-1+nuu-nu\sqrt{(1+uu)}) \\ +(n-1)(n-2)\beta u^{n-4}(n-3+(n-2)uu- \\ \quad (n-2)u\sqrt{(1+uu)})\end{array}\right\}(\sqrt{(1+uu)}+u)^n$$

po-

ponamus $\beta = \frac{1}{(n-2)(n-3)}$ & $\alpha = \frac{4(n-1)}{(n-2)(n-3)}$, ut ſit

$x = u^{n-4}$ erit:

$$(n-2)(n-3)y = u^{n-2}((n-2)(n-3) + nnuu + 4nu^{4} - u^{2}((n-2) + 4nuu)\sqrt{(1+uu)})(\sqrt{(1+uu)} + u)^{n}$$

Quod ſi jam ponatur $n = 5$, fiet $x = u$ ideoque

$$6y = x(6 + 25xx + 20x^{4} - (9x + 20x^{3})\sqrt{(1+xx)})(\sqrt{(1+xx)} + x)^{5}$$

Simili autem modo hic ita adjungi poteſt caſus $\lambda = n - 4$, ut poſito $n = 7$ fiat $x = u$, atque pari modo, quousque libuerit, progredi licet.

35. Ex reliquis formulis §. 9 exhibitis, in quibus tam x quam y per tertiam variabilem t, cujus u eſt functio impar, v vero functio par, exprimitur, in genere formulæ algebraicæ commode erui non poſſunt: ſin autem in his formulis $dx = vdt$ & $dy = \left(\frac{1+u}{1-u}\right)^{n} vdt$ ponatur $v = 1 - uu$, & $n = 1$, ut fiat $dx = (1 - uu)dt$ & $dy\,(1 + u)^{2}\,dt$, quæ formulæ non obſtante hac determinatione ſunt maxime generales. Cum igitur hinc ſit:

$$x = t - \int uudt \;\&\; y = t + 2\int udt + \int uudt$$

perſpicuum eſt curvas algebraicas prodire, ſi tam $\int udt$ quam $\int uudt$ fuerit integrabile. Ad hoc utrumque præſtandum

 in

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in formula: $\int u dt = tu - \int t du$ | ponatur $\int t du = p$; ut ſit $t = \frac{dp}{du}$

ubi, quia t eſt functio impar ipſius u, neceſſe eſt ut ſit p functio par. Jam hic valor ipſius t in altera formula ſubſtituatur, eritque

$$\int uudt = tuu - 2\int tudu = \frac{uudp}{du} - 2\int udp = \frac{uudp}{du} - 2pu + 2\int pdu$$

ponatur $\int pdu = q$, ut ſit $p = \frac{dq}{du}$, eritque q functio impar ipſius u; & ſumto du conſtante ſiet $dp = \frac{ddq}{du}$ atque ſubſtitutionibus retro factis erit $t = \frac{ddq}{du^2}$ &

$$\int uudt = \frac{uuddq}{du^2} - \frac{2udq}{du} + 2q \ \& \int udt = \frac{uddq}{du^2} - \frac{dq}{du}$$

37. His valoribus ſubſtitutis habebimus

Quartam Regulam pro inveniendis trajectoriis reciprocis algebraicis:

Sumatur q *functio quaecunque impar ipſius* u, *poſitoque elemento* du *conſtante erit curvae quaeſitae*

$$\text{Abſciſſa AP} = x = \frac{ddq(1 - uu)}{du^2} + \frac{2udq}{du} - 2q$$

$$\text{Applicata PM} = y = \frac{ddq(1 + u)^2}{du^2} - \frac{2dq(1 + u)}{du} + 2q$$

Ponamus $q = u^\lambda$, exiſtente λ numero impari, erit $x =$

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$$x = \lambda(\lambda-1)u^{\lambda-2}(1-uu) + 2\lambda u^{\lambda} - 2u^{\lambda}$$

$$y = \lambda(\lambda-1)u^{\lambda-2}(1+u)^2 - 2\lambda u^{\lambda-1}(1+u) + 2u^{\lambda}$$

ſive

$$x = \lambda(\lambda-1)u^{\lambda-2} - (\lambda-1)(\lambda-2)u^{\lambda}$$

$$y = \lambda(\lambda-1)u^{\lambda-2} + 2\lambda(\lambda-2)u^{\lambda-2} + (\lambda-1)(\lambda-2)u^{\lambda}$$

38. Ex his formulis variabilis u non difficulter eliminatur, ſi enim altera per alteram dividatur, erit

$$\lambda(\lambda-1)(x-y) + 2\lambda(\lambda-2)xu + (\lambda-1)(\lambda-2)uu(x+y) = 0$$

hincque $uu = \frac{-2\lambda(\lambda-2)xu - \lambda(\lambda-1)(x-y)}{(\lambda-1)(\lambda-2)(x+y)}$;
inveniturque

$$u = \frac{-\lambda(\lambda-2)x \pm \sqrt{\lambda(\lambda-2)((\lambda-1)^2 yy - xx)}}{(\lambda-1)(\lambda-2)(x+y)}$$

qui valor ſi in altera æquatione, vel in ſumma amborum $x+y = 2\lambda u^{\lambda-2}(\lambda-1+(\lambda-2)u)$ ſubſtituatur, orietur æquatio inter x & y, quæ ſemel tantum quadratis ſumendis ad rationalitatem reducetur. Quatuor autem hiſce regulis pro inventione trajectoriarum reciprocarum algebraicarum traditis, quicquid in ſolutione hujus problematis adhuc deſiderari poterat, hic abunde præſtitiſſe mihi videor.

DE

DE MOTU
Corporum Flexibilium.

I.

Tabb. III. & IV. Si duo copora rigida ita inter ſe conjungantur, ut utrumque circa juncturam libere moveri poſſit, ea invicem flexura connecti dicuntur. Axis autem flexuræ vocatur linea recta, circa quam ambo corpora libere gyrari queant. Si in extremitate alterius corporis tertium ſimili flexura coaptetur, tria habebuntur corpora duabus flexuris inter ſe connexa, quatuor autem corpora tribus flexuris connectentur, & ita porro. Hujusmodi corpus flexibile pluribus flexuris inſtructum catena repræſentat, cujus ſinguli articuli flexuris inter ſe ſunt connexi, numerusque flexurarum unitate deficiet a numero articulorum catenam conſtituentium. Funis autem & filum, ſi ſint perfecte flexibilia, conſiderari poſſunt, tanquam conſtarent ex pluribus minimis articulis flexuris inter ſe connexis. Hinc ope fili plurima corpora rigida ita invicem colligari poſſunt, ut corpus flexibile conſtituant. Hocque caſu cum partes quaquaverſus inter ſe flecti queant, quævis linea recta per flexuram ad fili connectentis directionem normalis locum axis flexuræ tenere poterit: ejus vero tantum ratio erit habenda, circa quem motus actu abſolvitur.

2. Ad motum ergo hujusmodi corporum flexibilium definiendum, ſingulorum primo articulorum motus inveſtigari debet. Deinde cum flexuræ impediant, quo minus partes a ſe invicem disjungantur, manifeſtum eſt hos ſingu-

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ſingularum partium motus quodammodo a ſe invicem pendere. Binorum enim quorumque articulorum extremitates, quæ flexuris inter ſe ſunt connexæ, perpetuo motum communem habere debent; ipſi vero articuli circa hanc flexuram motu angulari movebuntur. Hic igitur primum ipſarum ſingularum flexurarum motus ſunt conſiderandi, qui etſi in infinitum variare poſſunt, tamen hac lege inter ſe conſtringuntur, ut binæ contiguæ perpetuo æquali intervallo a ſe invicem diſtent. Hæc igitur motuum multiplicitas problema ſolutu difficillimum reddere videtur; interim tamen operam dabo, ut quantum fieri licet, hujus problematis tantopere complicati ſolutionem planam ac facilem exhibeam; quod commodiſſime fieri poterit, ſi a caſibus ſimpliciſſimis inveſtigationem ordiamur. Primum ergo unius articuli ſolitarii, qui cum aliis omnino non ſit connexus, motum determinabo.

Problema. I.

3. *Determinare motum virgae rigidae* AB *ſuper plano horizontali utcunque projectae.* Fig. 1.

Solutio.

Pervenerit hæc virga tempore quocunque t elapſo in ſitum AB, ad quem definiendum pro lubitu aſſumatur in eodem plano horizontali linea recta fixa OP pro axe, ad quem ex virgæ terminis A & B demittantur perpendicula Aa, Bb. Sit G centrum gravitatis virgæ AB, unde pariter ad axem normalis ducatur GP, ponaturque OP $= p$; PG $= x$, & angulus AGP $= \zeta$. Quod ſi ergo ad quodvis tempus determinare noverimus valores litterarum $p, x,$ & ζ, habebimus non ſolum locum & poſitionem virgæ AB,

 ſed

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ſed etiam ejus motum verum. Quodſi enim motus puncti G reſolvatur in binos laterales, quorum alterius directio ſit parallela lineæ OP, alterius vero incidat in ipſam rectam PG, erit celeritas illius $= \frac{dp}{dt}$; hujus vero $= \frac{dx}{dt}$, denotante dt tempuſculum, quo variabiles p & x augmenta accipiunt dp & dx. Cognito autem motu centri gravitatis G, motus virgæ angularis circa punctum G celeritas erit $= \frac{d\zeta}{dt}$; ſi quidem angulum AGP $= \zeta$ motu angulari augeri ponamus; ſcilicet $\frac{d\zeta}{dt}$ definiet celeritatem, qua virgæ punctum, quod a centro gravitatis G intervallo $= 1$ circa G gyratur. Quoniam hanc virgam a nullis viribus ſollicitari ponimus, atque planum horizontale, ſuper quo ſit motus, omni aſperitate deſtitutam aſſumimus, tam motus centri gravitatis, quam motus angularis virgæ circa centrum gravitatis erit uniformis, uti ex mechanicis conſtat. Erit ergo $\frac{dp}{dt} = \mathfrak{A}$, $\frac{dx}{dt} = \mathfrak{B}$ & $\frac{d\zeta}{dt} = \mathfrak{G}$; unde fit $p = \mathfrak{A}t + a$, $x = \mathfrak{B}t + b$ & $\zeta = \mathfrak{G}t + g$. Ex quibus æquationibus locus & poſitio virgæ ad quodvis tempus definietur, hincque ſimul ejus motus innoteſcit. Q. E. I.

Coroll. 1.

4. Ponamus motus initio, cum eſſet $t = 0$, centrum gravitatis G in O eſſe verſatum, directionemque virgæ AB ad rectam OP fuiſſe normalem, ita ut tum angulus ζ fuerit

fuerit $=0$, erit $a=0$, $b=0$, $g=0$, ideoque hoc caſu erit $p=\mathfrak{A}t$; $x=\mathfrak{B}t$; & $\zeta=\mathfrak{G}t$.

Coroll. 2.

7. Si præterea aſſumamus initio motus celeritatem centri gravitatis fuiſſe debitam altitudini $=a$, ejusque directionem incidiſſe in axem OP, erit $\mathfrak{A}=\sqrt{a}$ & $\mathfrak{B}=0$. fiet ergo $p=t\sqrt{a}$ & $x=0$, unde conſtat centrum gravitatis G perpetuo in axe OP motu uniformi eſſe progreſſurum, & celeritatem rotatoriam fore conſtantem.

Scholion.

6. Planiſſima hæc ſunt ex principiis mechanicæ, quibus conſtat, omne corpus rigidum, quod a nullis viribus ſollicitatur conſtanter ita moveri, ut ejus centrum gravitatis motu æquabili lineam rectam deſcribat, ſingulæque ejus partes circa centrum gravitatis motu uniformi rotentur. Interim tamen hoc problema præmittere viſum eſt, ut ex modo ſolutionis via ad ſequentia reſolvenda planior redderetur. Ceterum hinc jam motus definiri poteſt duorum corpuſculorum minimorum filo connexorum; ſi enim corpuſcula ſint minima, eorum motus, quo forteutrumque circa ſuum centrum gravitatis rotatur, negligi poteſt: movebunturque ambo, quamdiu filum tenſum manet, inſtar virgæ rigidæ, quemadmodum ex ſolutione ſequentis problematis perſpicietur.

Problema. II.

7. *Duorum corpuſculorum A et B filo inertiae experte AB inter ſe colligatorum motum ſuper plano horizontali determinare, poſtquam utcunque fuerint projecta.* Fig. 2.

 Solu-

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Solutio.

Sumta recta O ab pro axe, pervenerint corpuſcula hæc elapſo tempore t in ſitum AB, ex quibus ad axem perpendicula ducantur Aa & Bb. Sit longitudo fili AB $= a$, quæ perpetuo eadem manet; & vocetur corpuſculi A maſſa $=$ A, corpuſculi B maſſa $=$ B; ſintque O$a = p$; A$a = x$; O$b = q$ & B$b = y$; angulus vero ABb ponatur $= \zeta$; eritque

$$ab = q - p = a \operatorname{ſin} \zeta \ \& \ y - x = a \operatorname{coſ} \zeta$$

Tum vero erit celeritas corpuſculi A ſecundum directionem O$a = \frac{dp}{dt}$, & ſecundum directionem aA $= \frac{dx}{dt}$: ſimilique modo corpuſculi B celeritas ſecundum directionem Ob erit $= \frac{dq}{dt}$ & ſecundum directionem bB $= \frac{dy}{dt}$. Exprimat jam P tenſionem fili AB, qua vi corpus A verſus B, at corpus B verſus A trahitur. Corporis ergo A

motus $\frac{dp}{dt}$ accelerabitur vi $=$ P ſin ζ;

motus $\frac{dx}{dt}$ accelerabitur vi $=$ P coſ ζ; Corporis vero B

motus $\frac{dq}{dt}$ retardabitur vi $=$ P ſin ζ;

motus $\frac{dy}{dt}$ retardabitur vi $=$ P coſ ζ. Hinc erit ex natura ſollicitationum, poſito elemento dt conſtante:

$$2\,A\,ddp = P\,dt^2 \operatorname{ſin} \zeta; \quad 2\,A\,ddx = P\,dt^2 \operatorname{coſ} \zeta$$

$$2\,B\,ddq = -P\,dt^2 \operatorname{ſin} \zeta; \quad 2\,B\,ddy = -P\,dt^2 \operatorname{coſ} \zeta.$$

Ex

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Ex his erit $A ddp + B ddq = 0$; & bis integrando $Ap + Bq = \mathfrak{A}t + \mathfrak{a}$, ſimilique modo erit $A ddx + B ddy = 0$ unde $Ax + By = \mathfrak{B}t + \mathfrak{b}$. Cum ergo $q - p = a \sin \zeta$ erit $p = \frac{\mathfrak{A}t + \mathfrak{a} - Ba \sin \zeta}{A + B}$ & $q = \frac{\mathfrak{A}t + \mathfrak{a} + Aa \sin \zeta}{A + B}$. Atque ob $y - x = a \cos \zeta$, erit $x = \frac{\mathfrak{B}t + \mathfrak{b} - Ba \cos \zeta}{A + B}$ & $y = \frac{\mathfrak{B}t + \mathfrak{b} + Aa \cos \zeta}{A + B}$.

Deinde vero cum ſit $\frac{ddp}{ddx} = \frac{\sin \zeta}{\cos \zeta}$, ob $ddp = -\frac{Ba\, dd.\sin \zeta}{A+B}$ & $ddx = -\frac{Ba\, dd.\cos \zeta}{A+B}$, erit $\frac{dd.\sin \zeta}{dd.\cos \zeta} = \frac{\sin \zeta}{\cos \zeta}$, ſeu $\sin \zeta . dd.\cos \zeta - \cos \zeta . dd \sin \zeta = 0$ cujus integrale eſt
$-\sin \zeta . d \cos. \zeta + \cos \zeta . d \sin \zeta = \mathfrak{G} dt$

ſeu $+ d\zeta . \sin \zeta^2 + d\zeta . \cos \zeta^2 = + d\zeta = \mathfrak{G} dt$; unde fit $\zeta = \mathfrak{G}t + \mathfrak{g}$. Quocirca ad quodvis tempus t definietur angulus $ABb = \zeta$, hincque valores litterarum v, x, q & y ſicque poſitio corpuſculorum A & B filo inter ſe connexorum perpetuo aſſignari poterit. Q. E. J.

Coroll. 1.

8. Quia eſt $A dp + B dq = \mathfrak{A} dt$ ſeu $\frac{A dp + B dy}{(A + B) dt} = \frac{\mathfrak{A}}{A + B}$, centrum commune gravitatis amborum corporum ſecundum directionem axis Oab uniformiter progreditur;

tur; ſimili vero modo ob $\frac{A dx + B dy}{(A+B) dt} = \frac{\mathfrak{B}}{A+B}$, quoque ab hoc axe motu uniformi recedet, ideoque conjunctim motu uniformi lineam rectam deſcribet.

Coroll. 2.

9. Quoniam eſt $\frac{d\zeta}{dt} = \mathfrak{G}$ ac propterea conſtans, ſequitur angulum A B b uniformiter increſcere, ideoque filum AB uniformiter in gyrum agi; atque adeo hæc duo corpuſcula A & B filo AB connexa perinde movebuntur, ac ſi virga rigida in ipſorum locum ſubſtitueretur.

Coroll. 3.

10. Cum ſit $\zeta = \mathfrak{G} t + g$ & $d\zeta = \mathfrak{G} dt$, erit $d.$ ſin $\zeta = \mathfrak{G} dt$ coſ ζ, & $dd.$ ſin $\zeta = - \mathfrak{G}^2 dt^2$ ſin ζ: unde $ddp = + \frac{Ba \mathfrak{G}^2 dt^2 \text{ ſin } \zeta}{A+B}$. Quare cum ſit $2 A ddp = P dt^2$ ſin ζ, fiet $\frac{2 A B a \mathfrak{G}^2}{A+B} = P$, quæ eſt vis qua filum tenditur: ubi notandum eſt, $\mathfrak{G} = \frac{d\zeta}{dt}$ definire celeritatem rotatoriam fili AB ad diſtantiam $= 1$ relatam. Hinc ſi f ſit altitudo debita celeritati, qua A circa B & viciſſim B circa A revolvitur, erit $\mathfrak{G} = \frac{\sqrt{f}}{a}$; fietque tenſio fili $P = \frac{2 A B}{A+B} \cdot \frac{f}{a}$.

Scho-

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Scholion.

11. Etſi ſolutio hujus problematis facillime ex mechanicæ principiis deduci potuiſſet, tamen hæc ſolutio, quamvis ſit prolixior, quam natura rei poſtulat, hunc nobis præſtabit uſum, ut eadem methodo caſus, ſi tria & plura corpuſcula filo fuerint colligata, evolvi queant, quod fieri non poſſet, ſi ſolutionem maxime naturalem & concinnam adhibere voluiſſemus. Progrediar ergo ad caſum trium corporum examinandum, quem Vir Celeb. Daniel Bernoulli mihi propoſuit, & in quo fundamentum univerſæ theoriæ de motu corporum flexibilium eſt poſitum. Hoc enim caſu expedito non difficile erit, eandem methodum ad plura corpuſcula atque ad omnia corpora flexibilia extendere.

Problema. III.

12. *Si tria corpuſcula A, B, C filo inertiae experte fuerint connexa, eaque ſuper plano horizontali utcunque projiciantur, eorum motum determinare.* Fig. 3.

Solutio.

Sumta pro lubitu recta Oo pro axe fixo, pervenerint corpuſcula elapſo tempore t in ſitum ABC; ductisque ad axem perpendiculis ſit $Oa = p$, $Aa = x$; $Ob = q$; $Bb = y$; $Oc = r$, $Cc = z$; tum vero ponatur $AB = a$, $BC = b$, ang. $ABb = \zeta$; ang. $BCc = \eta$, eritque.

$$q - p = a \sin \zeta; \quad r - q = b \sin \eta$$
$$y - x = a \cos \zeta; \quad z - y = b \cos \eta$$

Ex his motus ſingulorum corpuſculorum ita definientur, ut poſito elemento temporis $= dt$, ſit

Cele-

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Celeritas	in direčtione Oo	Celeritas	in direčtione $O\omega$
Corpuſculi A	$= \frac{dp}{dt}$	Corpuſculi A	$= \frac{dx}{dt}$
Corpuſculi B	$= \frac{dq}{dt}$	Corpuſculi B	$= \frac{dy}{dt}$
Corpuſculi C	$= \frac{dr}{dt}$	Corpuſculi C	$= \frac{dz}{dt}$

Denotent jam litteræ A, B, C maſſas corpuſculorum A, B, & C, ſitque P tenſio fili AB, Q tenſio fili BC; unde ſequentes orientur ſingulorum corpuſculorum ſollicitationes.

Sollicitabitur	in direčtione Oo	in direčtione $O\omega$
Corpuſculum A	vi $= P \sin \zeta$	vi $= P \cos \zeta$
Corpuſculum B	vi $= - P \sin \zeta + Q \sin \eta$	vi $= - P \cos \zeta + Q \cos \eta$
Corpuſculum C	vi $= - Q \sin \eta$	vi $= - Q \cos \eta$

Ad incrementa velocitatum definienda ponamus corpuſculi A celeritatem in direčtione Oo debitam eſſe altitudini v, & cum ſollicitetur a vi $= P \sin \zeta$, dum ſpatiolum dp abſolvit, fiet $dv = \frac{P\,dp \sin \zeta}{A}$, uti ex principiis mechanicæ conſtat. Quia autem celeritas eſt $= \frac{dp}{dt}$ faciamus $\frac{dp}{dt} = \sqrt{v}$, erit $v = \frac{dp^2}{dt^2}$, & poſito elemento temporis dt conſtante erit $dv = \frac{2\,dp\,ddp}{dt^2}$, quo valore ſubſtituto habebimus $\frac{2\,dp\,ddp}{dt^2} = \frac{P\,dp \sin \zeta}{A}$ ſeu $2\,A\,ddp = P\,dt^2 \sin \zeta$.

Hoc

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Hoc igitur modo ſingulorum corpuſculorum accelerationes deſiniendo obtinebimus ſequentes æquationes.

$$2\,A\,ddp = P\,dt^2 \text{ſin}\,\zeta; \qquad 2\,A\,ddx = P\,dt^2 \text{coſ}\,\zeta$$

$$2\,B\,ddq = -P\,dt^2 \text{ſin}\,\zeta + Q\,dt^2 \text{ſin}\,\eta \qquad 2\,B\,ddy = -P\,dt^2 \text{coſ}\,\zeta + Q\,dt^2 \text{coſ}\,\eta$$

$$2\,C\,ddr = -Q\,dt^2 \text{ſin}\,\eta; \qquad 2\,C\,ddz = -Q\,dt^2 \text{coſ}\,\eta$$

Ex his æquationibus additis naſcuntur duæ ſequentes;

$$2\,A\,ddp + 2B\,ddq + 2\,C\,ddr = 0$$
$$2\,A\,ddx + 2B\,ddy + 2\,C\,ddz = 0$$

quæ bis integratæ dant:

$$2\,A\,p + B\,q + C\,r = \mathfrak{A}t + \mathfrak{a}$$
$$2\,A\,x + B\,y + C\,z = \mathfrak{B}t + \mathfrak{b}$$

Cum vero ſit $q = p + a\,\text{ſin}\,\zeta$; $y = x + a\,\text{coſ}\,\zeta$; atque porro $r = p + a\,\text{ſin}\,\zeta + b\,\text{ſin}\,\eta$ & $z = x + a\,\text{coſ}\,\zeta + b\,\text{coſ}\,\eta$, erit his valoribus ſubſtitutis:

$$(A+B+C)\,p + (B+C)\,a\,\text{ſin}\,\zeta + C\,b\,\text{ſin}\,\eta = \mathfrak{A}t + \mathfrak{a} \quad \&$$
$$(A+B+C)\,x + (B+C)\,a\,\text{coſ}\,\zeta + C\,b\,\text{coſ}\,\eta = \mathfrak{B}t + \mathfrak{b}$$

ex quibus orientur ſequentes determinationes:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} - (B+C)\,a\,\text{ſin}\,\zeta - C\,b\,\text{ſin}\,\eta}{A+B+C}$$

$$q = \frac{\mathfrak{A}t + \mathfrak{a} + A\,a\,\text{ſin}\,\zeta - C\,b - \text{ſin}\,\eta}{A+B+C}$$

$$r = \frac{\mathfrak{A}t + \mathfrak{a} + A\,a\,\text{ſin}\,\zeta + (A+B)\,b\,\text{ſin}\,\eta}{A+B+C}$$

 ſimi-

ſimilique modo reperietur:

$$x = \frac{\mathfrak{B}t + \mathfrak{b} - (B + C)\,a\cos\zeta - Cb\cos\eta}{A + B + C}$$

$$y = \frac{\mathfrak{B}t + \mathfrak{b} + Aa\cos\zeta - Cb\cos\eta}{A + B + C}$$

$$z = \frac{\mathfrak{B}t + \mathfrak{b} + Aa\cos\zeta + (A + B)\,b\cos\eta}{A + B + C}$$

Inventis litteris $p, q, r,$ & $x, y, z,$ tenſiones P & Q duplici-modo exprimentur. Erit enim

$$P\,dt^2 = \frac{2\,A\,ddp}{\sin\zeta} \quad \& \quad P\,dt^2 = \frac{2\,A\,ddx}{\cos\zeta}$$

$$Q\,dt^2 = \frac{-2\,C\,ddr}{\sin\eta} \quad \& \quad Q\,dt^2 = \frac{-2\,C\,ddz}{\cos\eta}$$

unde fit:

$$ddp\cos\zeta = ddx\sin\zeta \quad \& \quad ddr\cos\eta = ddz\sin\eta$$

ſubſtitutis autem valoribus ante inventis . . .

$$(B + C)\,a\cos\zeta\,dd.\sin\zeta + Cb\cos\zeta\,dd.\sin\eta = (B + C)\,a\sin\zeta\,dd.\cos\zeta + Cb\sin\zeta\,dd.\cos\eta$$

ſeu $(B + C)\,a(\cos\zeta\,dd.\sin\zeta - \sin\zeta\,dd.\cos\zeta) + Cb(\cos\zeta\,dd.\sin\eta - \sin\zeta\,dd.\cos\eta) = 0$

& $Aa(\cos\eta\,dd.\sin\zeta - \sin\eta\,dd.\cos.\zeta) + (A + B)\,b(\cos\eta\,dd.\sin\eta - \sin\eta\,dd.\cos\eta) = 0$.

At vero generatim eſt $\cos m\,dd.\sin n - \sin m\,dd.\cos n = \cos m(ddn\cos n - dn^2\sin n) + \sin m(ddn\sin n + dn^2\cos n)$, ideoque cum ſit $\cos m\cos n + \sin m\sin n = \cos(m - n)$ & $\sin m\cos n - \cos m\sin n = \sin(m - n)$ erit

cos

$\operatorname{coſ} m\, dd.\ \operatorname{ſin} n - \operatorname{ſin} m.\ dd \operatorname{coſ} n = ddn \operatorname{coſ}(m-n) + dn^2 \operatorname{ſin}(m-n)$

Cujus reductionis ratione habita fiet

$$(B+C)\, a\, dd\zeta + C\, b\, dd\eta \operatorname{coſ}(\zeta-\eta) + C\, b\, d\eta^2 \operatorname{ſin}(\zeta-\eta) = 0$$

$$(A+B)\, b\, dd\eta + A\, a\, dd\zeta \operatorname{coſ}(\eta-\zeta) + A\, a\, d\zeta^2 \operatorname{ſin}(\eta-\zeta) = 0$$

Integretur utraque quoad fieri poteſt, erit

$$(B+C)\, a\, d\zeta + C\, b\, d\eta \operatorname{coſ}(\zeta-\eta) + C\, b \int d\zeta\, d\eta \operatorname{ſin}(\zeta-\eta) = \text{Conſt.}$$

$$(A+B)\, b\, dy + A\, a\, d\zeta \operatorname{coſ}(\eta-\zeta) + A\, a \int d\zeta\, d\eta \operatorname{ſin}(\eta-\zeta) = \text{Conſt.}$$

Unde partibus integralibus eliminatis fiet:

$$\frac{(B+C)\, a\, d\zeta}{Cb} + d\eta \operatorname{coſ}(\zeta-\eta) + \frac{(A+B)\, b\, dy}{Aa} + d\zeta \operatorname{coſ}(\eta-\zeta) = \text{Conſt.}$$

ſeu $\dfrac{(B+C)\, a\, d\zeta}{Cb} + \dfrac{(A+B)\, b\, d\eta}{Aa} + (d\zeta + d\eta) \operatorname{coſ}(\zeta-\eta) = \dfrac{dt}{\sqrt{f}}$

Per ſubtractionem vero ex duabus illis æquationibus differentialibus oritur:

$$\frac{(B+C)\, a\, dd\zeta}{C\, b} - \frac{(A+B)\, b\, dd\eta}{A\, a} - (dd\zeta - dd\eta) \operatorname{coſ}(\zeta-\eta) + (d\zeta^2 + d\eta^2) \operatorname{ſin}(\zeta-\eta) = 0$$

Ponatur $\zeta + \eta = v$ & $\zeta - \eta = u$, ut ſit $\zeta = \dfrac{v+u}{2}$

& $\eta = \dfrac{v-u}{2}$

abibitque prior æquatio integrata in hanc:

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$$\frac{(B+C)adv}{2Cb}+\frac{(B+C)adu}{2Cb}+\frac{(A+B)bdv}{2Aa}-\frac{(A+B)bdu}{2Aa}$$

$$+dv\,\mathrm{coſ}\,u=\frac{dt}{\sqrt{f}}$$

Poſterior vero æquatio differentio differentialis in hanc:

$$\frac{(B+C)addv}{2Cb}+\frac{(B+C)addu}{2Cb}-\frac{(A+C)bddv}{2Aa}+$$

$$\frac{(A+B)bddu}{2Aa}-ddu\,\mathrm{coſ}\,u+\tfrac{1}{2}(dv^2+du^2)\,\mathrm{ſin}\,u=0$$

Ponatur brevitatis gratia:

$$\frac{(B+C)a}{C}+\frac{(A+B)b}{Aa}=m$$

$$\frac{(B+C)a}{Cb}-\frac{(A+B)b}{Aa}=n$$

orienturque iſtæ duæ æquationes:

$$\tfrac{1}{2}mdv+\tfrac{1}{2}ndu+dv\,\mathrm{coſ}\,u=\frac{dt}{\sqrt{f}}$$

$$\tfrac{1}{2}nddv+\tfrac{1}{2}mddu-ddu\,\mathrm{coſ}\,u+\tfrac{1}{2}(dv^2+du^2)\,\mathrm{ſin}\,u=0$$

Ex priori fit $dv=\frac{2dt:\sqrt{f}-ndu}{m+2\,\mathrm{coſ}\,u}$, qui in altera

$$nddv+mddu-2ddu\,\mathrm{coſ}\,u+(dv^2+du^2)\,\mathrm{ſin}\,u=0$$

ſubſtitutus ob $ddv=\frac{-nddu}{m+2\,\mathrm{coſ}\,u}+\frac{4dtdu\,\mathrm{ſin}\,u:\sqrt{f}-2ndu^2\,\mathrm{ſin}\,u}{(m+2\,\mathrm{coſ}\,u)^2}$

dabit:

$-nn$

$$\frac{-nnddu}{m+2\cos u} = \frac{4ndtdu\sin u : \sqrt{f} - 2nndu^2\sin u}{(m+2\cos u)^2} + mddu$$

$$-2ddu\cos u + du^2\sin u + \frac{4dt\sin u : f - 4ndtdu\sin u : \sqrt{f} + nndu^2\sin u}{(m+2\cos u)^2} = 0$$

ſeu $(mm-nn)ddu - 4ddu\cos u^2 + \frac{4dt^2\sin u : f + (mm-nn)du^2\sin u}{m+2\cos u}$

$$+\frac{4mdu^2\sin u\cos u + 4du^2\sin u\cos u^2}{m+2\cos u} = 0$$

Ponatur $dt = \frac{du}{\sqrt{\omega}}$ erit $ddt = 0 = \frac{ddu}{\sqrt{\omega}} - \frac{du\,d\omega}{2\omega\sqrt{\omega}}$ ideoque $ddu = \frac{du\,d\omega}{2\omega}$, quibus valoribus pro dt & ddu ſubſtitutis erit:

$$\frac{(mm-nn)d\omega}{2\omega} - \frac{2d\omega\cos u^2}{\omega} + \frac{4du\sin u : f\omega + (mm-nn)du\sin u}{m+2\cos u}$$

$$+\frac{4mdu\sin u\cos u + 4du\sin u\cos u^2}{m+2\cos u} = 0 \text{ ſeu}$$

$$\tfrac{1}{2}(m^2 - n^2)\,d.\omega - 2d\omega\cos u^2 - \frac{m\omega du\sin u}{m+2\cos u}$$

$$+(m+2\cos u)\omega du\sin u + \frac{4du\sin u}{f(m+2\cos u)} = 0$$

Sit $\operatorname{coſ} u = s$ atque ob $du \operatorname{ſin} u = -ds$ erit:

$$(m^2-n^2)d\omega - 4ssd\omega + \frac{2nn\omega ds}{m+2s} - 2(m+2s)\omega ds = \frac{8ds}{f(m+2s)}$$

ſeu
$$d\omega + \frac{2\omega ds(nn-(m+2s)^2)}{(m+2s)(mm-nn-4ss)} = \frac{8ds}{f(m+2s)(m^2-n^2-4ss)}$$

quæ æquatio multiplicata per $\frac{mm-m-4ss}{m+2s}$ fit integrabilis, eritque integrale.

$$\frac{(mm-nn-4ss)\omega}{m+2s} = \int\frac{8ds}{f(m+2s)^2} = \frac{4}{g} - \frac{4}{f(m+2s)}$$

hinc erit $\omega = \frac{4f(m+2s)-4g}{fg(mm-nn-4ss)} = \frac{du^2}{dt^2}$ & ob $du = \frac{-ds}{\sqrt{(1-ss)}}$ fiet $dt = \frac{-ds\sqrt{fg(mm-nn-4ss)}}{2\sqrt{(1-ss)(mf-g+2fs)}}$ ſeu

$$t = \int\frac{ds\sqrt{fg(mm-nn-4ss)}}{2\sqrt{(mf-g+2fs)(1-ss)}}$$ vel etiam

$$t = \int\frac{du\sqrt{fg(mm-nn-4\operatorname{coſ} u^2)}}{2\sqrt{(mf-g+2f\operatorname{coſ} u)}}.$$ Hancobrem erit

$$\frac{2dt}{\sqrt f} = \frac{du\sqrt{g(mm-nn-4\operatorname{coſ} u^2)}}{\sqrt{(mf-g+2f\operatorname{coſ} u)}}$$ atque adeo

$$v = 2\int\frac{dt}{(m+2\operatorname{hoſ} u)\sqrt f} - \int\frac{ndu}{m+2\operatorname{coſ} u}.$$ Ex unica ergo variabili u, definiuntur t & v, ex hisque porro anguli ζ & η, quibus inventis facile aſſignantur coordinatæ p, q, r & x, y, z ſicque

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ſicque ad datum tempus poſitio corpuſculorum A, B, C determinari poterit. Q. E. I.

Coroll. 1.

13. Celeritas centri communis gravitatis ſecundum directionem axis Oo eſt $= \frac{A\,dp + B\,dq + C\,dr}{(A+B+C)\,dt} = \frac{\mathfrak{A}}{A+B+C}$; ideoque uniformis; ſimili modo ejus celeritas ſecundum directionem rectæ $O\omega$ normalis ad Oo eſt $= \frac{A\,dp + B\,dy + C\,dz}{(A+B+C)\,dt}$ $= \frac{\mathfrak{B}}{A+B+C}$, ideoque pariter uniformis, unde colligitur centrum commune gravitatis trium corpuſculorum A, B, & C uniformiter in directum progredi. Quodquidem a priori colligi potuiſſet ex natura omnium motuum ſibi relictorum.

Coroll. 2.

14. Si igitur fuerit & $\mathfrak{A}$ & $\mathfrak{B} = 0$, tum commune centrum gravitatis quieſcet. Quodſi ergo toti ſyſtemati imprimatur motus æqualis & contrarius motui centri gravitis tum id revera quieſcet, fietque propterea $\mathfrak{A} = 0$, & $\mathfrak{B} = 0$.

Coroll. 3.

15. Quoniam hoc caſu, quo centrum gravitatis quieſcit & proinde fit $\mathfrak{A} = 0$, $\mathfrak{B} = 0$, ejus diſtantia in recta $Oo\,a$ puncto O eſt $= \frac{Ap + Bq + Cr}{A+B+C} = \frac{a}{A+B+C}$, & diſtan-

diſtantia ſecundum $O\omega$ a puncto O eſt $= \frac{Ax+By+Cz}{A+B+C} = \frac{\mathfrak{b}}{A+B+C}$, ſi fuerit $\mathfrak{a} = 0$ & $\mathfrak{b} = 0$, tum centrum gravitatis in puncto O quieſcet.

Coroll. 4.

16. Cum ergo ob rectam Oo cum puncto O arbitrariam motus ſemper ad hunc caſum reduci poſſit, ut centrum gravitatis in puncto O quieſcat, nullam vim amplitudini ſolutionis inferendo ſemper licebit conſtantes $\mathfrak{A}$, $\mathfrak{B}$ & $\mathfrak{a}$, $\mathfrak{b}$ evaneſcentes aſſumere: eritque propterea:

$$p = -\frac{(B+C)a\sin\zeta - Cb\sin\eta}{A+B+C};\quad x = -\frac{(B+C)a\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$q = \frac{Aa\sin\zeta - Cb\sin\eta}{A+B+C};\quad y = \frac{Aa\cos\zeta - Cb\cos\eta}{A+B+C}$$

$$r = \frac{Aa\sin\zeta + (A+B)b\sin\eta}{A+B+C};\quad z = \frac{Aa\cos\zeta + (A+B)b\cos\eta}{A+B+C}$$

Coroll. 5.

17. Hinc porro celeritates corporum ſecundum utramque directionem Oo & $O\omega$ definiri poterunt:

Secun-

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Secundum directionem Oo	Secundum directionem Ou
$\frac{dp}{dt} = \frac{-(B+C)ad\zeta\cos\zeta - Cbd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dx}{dt} = \frac{(B+C)ad\zeta\sin\zeta + Cbd\eta\sin\eta}{(A+B+C)dt}$
$\frac{dq}{dt} = \frac{Aad\zeta\cos\zeta - Cbd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dy}{dt} = \frac{-Aad\zeta\sin\zeta + Cbd\eta\sin\eta}{(A+B+C)dt}$
$\frac{dr}{dt} = \frac{Aad\zeta\cos\zeta + (A+B)bd\eta\cos\eta}{(A+B+C)dt}$	$\frac{dz}{dt} = \frac{-Aad\zeta\sin\zeta - (A+B)bd\eta\sin\eta}{(A+B+C)dt}$

Coroll. 6.

18. Conſervatio virium vivarum ex ſollicitationibus momentaneis facile colligitur. Cum enim ſumma virium vivarum ſit $= \frac{Adp^2 + Adx^2 + Bdq^2 + Bdy^2 + Cdr^2 + Cdz^2}{dt^2}$ erit ejus differentiale $\frac{2Adpddp}{dt^2} + \frac{2Adxddx}{dt^2} + \frac{2Bdqddq}{dt^2}$

$$+ \frac{2Bdyddy}{dt^2} + \frac{2Cdrddr}{dt^2} + \frac{2Cdzddz}{dt^2}$$

$$= \left\{ \begin{array}{l} P(dp\sin\zeta + dx\cos\zeta - dq\sin\zeta - dy\cos\zeta) \\ Q(dq\sin\eta + dy\cos\eta - dr\sin\eta - dz\cos\eta) \end{array} \right. = 0.$$

Cum enim ſit $q - p = a\sin\zeta$, erit $dq - dp = ad.\sin\zeta$

et $(dp - dq) \text{ſin} \zeta = -\frac{1}{2} ad. \text{ ſin} \zeta^2$. Simili modo erit $(dx - dy) \text{coſ} \zeta = -\frac{1}{2} ad. \text{coſ} \zeta^2 = + \frac{1}{2} ad. \text{ſin} \zeta^2$ ob $\text{coſ} \zeta^2, = 1 - \text{ſin} \zeta^2$: ideoque coëfficiens ipſius P eſt $= 0$, pariterque ipſius Q. Unde differentiale virium vivarum eſt $= 0$, ideoque ſumma virium vivarum conſtans.

Coroll. 7.

19. Ex iisdem æquationibus ſolutiones momentaneas exprimentibus concluditur fore:

$$\frac{2\,A x ddp + 2 B y ddq + 2\,C z ddr}{d t^2} = P(x - y) \text{ſin} \zeta + Q(y - z) \text{ſin} \eta =$$

$- P a \text{ ſin} \zeta \text{ coſ} \zeta - Q b \text{ ſin} \eta \text{ coſ} \eta$, ſimilique modo:

$$\frac{2\,A p ddx + 2 B q ddy + 2\,C r ddz}{d t^2} = P(p - q) \text{coſ} \zeta + Q(q - r) \text{coſ} \eta =$$

$- P a \text{ ſin} \zeta \text{ coſ} \zeta - Q b \text{ ſin} \eta \text{ coſ} \eta$, ex quibus ſequitur fore:

$A(xddp - pddx) + B(yddp - qddy) + C(zddr - rddz) = 0$

cujus integrale eſt:

$$A(xdp - pdx) + B(ydq - qdy) + C(zdr - rdz) = 0.$$

Coroll. 8.

20. Reliquæ conſtantes, quæ integratione in ſolutionem introducuntur, ex ſtatu corpuſculorum initiali determinari debent. Si igitur aſſumamus centrum gravitatis in O perpetuo quieſcere, atque copuſcula initio omnia in recta $O o$ ſita fuiſſe, anguli ζ & η ita definiri debent, ut poſito $t = 0$ fiant recti. Tum igitur celeritates corporum ſecundum axem $O o$ evaneſcent, at vero ſecundum $O \omega$ erunt, ut ſequitur

Fig. 4.

dx

$$\frac{dx}{dt} = \frac{(B+C)\,ad\zeta + Cbd\eta}{(A+B+C)\,dt};\quad \frac{dy}{dt} = \frac{-A\,ad\zeta + Cbd\eta}{(A+B+C)\,dt};$$

atque $\frac{dz}{dt} = \frac{-A\,ad\zeta - (A+B)\,bd\eta}{(A+B+C)\,dt}$.

Scholion.

21. Solutio ergo hujus problematis ab integratione formulæ $t = \int \frac{du\sqrt{fg(mm-nn-4\,\mathrm{coſ}\,u^2)}}{2\sqrt{(mf-g+2f\,\mathrm{coſ}\,u)}}$, & propterea per quadraturam curvæ cujuſpiam conſtrui poteſt, ita ut ad quemvis valorem ipſius t valor anguli u aſſignetur: quo autem invento nova opus eſt quadratura ad angulum v determinandum; quamobrem ſolutio practica hujus problematis maxime eſt operoſa. Dantur tamen nonnulli caſus, quibus ſolutio multo fit ſimplicior ac tractabilior, quos hic ſeorſim evolvamus.

Caſus. I.

22. Quoniam invenimus inter t & u hanc æquationem

$$2\,dt\sqrt{(mf-g+2f\,\mathrm{coſ}\,u)} = du\sqrt{fg(m^2-n^2-4\,\mathrm{coſ}\,u^2)}$$

manifeſtum eſt huic æquationi ſatisfieri, ſi fuerit:

$mf-g+2f\,\mathrm{coſ}\,u = 0$ ſeu $\mathrm{coſ}\,u = \frac{g-mf}{2f}$; fiet enim u conſtans, & $du = 0$, unde utrumque membrum evaneſcit: Sit igitur $u = 2\alpha$, ut ſit $\mathrm{coſ}\,2\alpha = \frac{g-mf}{2f}$, eritque $m + 2\,\mathrm{coſ}\,u = \frac{g}{f}$; ideoque $v = 2\int\frac{dt\sqrt{f}}{g}$, ac propterea $v =$

$\frac{2t\sqrt{f}}{g} + 2\beta$. Hinc fit $\zeta = \frac{t\sqrt{f}}{g} + \beta + \alpha$ & $\eta = \frac{t\sqrt{f}}{g} + \beta - \alpha$. Cum igitur differentia angulorum ζ & η ſit conſtans, angulus ABC perpetuo idem manebit, corpuſculaque A, B, C perinde movebuntur, ac ſi corpus inflexile conſtituerent. Si igitur ponamus centrum gravitatis perpetuo in puncto O quieſcere, caſus iſte locum habebit, ſi initio quo $t = 0$, corpuſcula ita fuerint collocata, ut eſſet:

$$p = \frac{-(B+C)a\sin(\beta+\alpha) - Cb\sin(\beta-\alpha)}{A+B+C}$$

$$q = \frac{+Aa\sin(\beta+\alpha) - Cb\sin(\beta-\alpha)}{A+B+C}$$

$$r = \frac{+Aa\sin(\beta+\alpha) + (A+B)b\sin(\beta-\alpha)}{A+B+C}$$

Atque
$$x = \frac{-(B+C)a\cos(\beta+\alpha) - Cb\cos(\beta-\alpha)}{A+B+C}$$

$$y = \frac{Aa\cos(\beta+\alpha) - Cb\cos(\beta-\alpha)}{A+B+C}$$

$$z = \frac{Aa\cos(\beta+\alpha) + (A+b)\cos(\beta-\alpha)}{A+B+C}$$

Perpetuo vero celeritates corpuſculorum ob $d\zeta = d\eta = \frac{dt\sqrt{f}}{g}$ ita ſe habebunt, ut ſit

$$\frac{dp}{dt} = \frac{x\sqrt{f}}{g};\ \frac{dq}{dt} = \frac{y\sqrt{f}}{g};\ \frac{dr}{dt} = \frac{z\sqrt{f}}{g}$$

xd

$$\frac{dx}{dt} = -\frac{p\sqrt{f}}{g};\quad \frac{dy}{dt} = -\frac{q\sqrt{f}}{g};\quad \frac{dz}{dt} = -\frac{r\sqrt{f}}{g}.$$

unde colliguntur diſtantiæ ſingulorum corpuſculorum tam inter ſe quam a puncto O conſtantes. Atque cum anguli ζ & η æqualiter & uniformiter creſcant, ſingula corpora circa centrum gravitatis O æquali motu rotatorio uniformiter gyrabuntur.

Ponamus initio motus omnia corpuſcula in linea recta Oo poſita fuiſſe, atque ob $x = o$, $y = o$, $z = o$, erit Fig. 4.

$\beta + \alpha = 90^{\circ}$ & $\beta - \alpha = 90^{\circ}$, unde $\alpha = o$, & $\beta = 90^{\circ}$. Iſte ergo caſus locum habebit ſi fuerit:

$$OA = \frac{(B+C)a + Cb}{A+B+C} = -p$$

$$OB = \frac{-Aa + Cb}{A+B+C} = -$$

$$OC = \frac{Aa + (A+B)b}{A+B+C} = r$$

atque ſi ſingulis corpuſculis ſecundum directionem $O\omega$ impreſſæ fuerint celeritates, quæ ſint inter ſe uti OA, OB, & OC, tum filum ABC circa punctum O inſtar virgæ rigidæ rotabitur motu uniformi.

Caſus. II.

23. Ponatur $f = \infty$ & $n = o$ ſeu $\frac{(B+C)a}{Cb} = \frac{(A+B)b}{Aa}$, eritque v quantitas conſtans, ſit ea $v = 2\alpha$, tum

 vero

vero erit $t = \int \frac{du\sqrt{g(m - 2\,\mathrm{coſ}\,u)}}{2}$, ex qua æquatione facilius ad datum tempus angulus u definiri poteſt. Cum igitur ſit $\zeta = \alpha + \frac{1}{2}u$ & $\eta = \alpha - \frac{1}{2}u$, quantum alter augetur, tantum alter diminuitur: hincque anguli ABb & CBb æqualiter perpetuo vel creſcent decreſcent. Si recta AB producatur in γ, erit angulus CB$\gamma = \zeta - \eta = u$, hicque perpetuo a tempore jam elapſo t ita pendebit, ut ſit $t = \frac{1}{2}\int du\sqrt{g(m - 2\,\mathrm{coſ}\,u)}$. Facilius autem hinc ex angulo CBγ tempus jam elapſum t determinari poterit. Ut vero ex angulo u poſitio omnium corpuſculorum definiri queat, notandum

eſt, quia $\frac{(B+C)a}{Ca} = \frac{(A+B)b}{Aa}$ fore

$$(B+C)\,a = \tfrac{1}{2}mCb \ \&\ (A+B)\,b = \tfrac{1}{2}mAa,$$

& $m = 2\sqrt{\frac{(A+B)(B+C)}{AC}}$. Hincque obtinebitur

$\frac{a}{b} = \sqrt{\frac{(A+B)C}{(B+C)A}}$; quod eſt requiſitum, ut præſens caſus locum habere poſſit. Erit ergo elapſo tempore t, quo angulus CB$\gamma = u$ eſt ortus:

$$p = \frac{-(A+C)a\,\mathrm{ſin}(\alpha + \frac{1}{2}u) - Cb\,\mathrm{ſin}(\alpha - \frac{1}{2}u)}{A+B+C}$$

$$q = \frac{Aa\,\mathrm{ſin}(\alpha + \frac{1}{2}u) - Cb\,\mathrm{ſin}(\alpha - \frac{1}{2}u)}{A+B+C}$$

$$r = \frac{Aa\,\mathrm{ſin}(\alpha + \frac{1}{2}u) + (A+B)b\,\mathrm{ſin}(\alpha - \frac{1}{2}u)}{A+B+C}$$

$x =$

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$$x = \frac{-(B+C)a\cos(\alpha+\frac{1}{2}u)-Cb\cos(\alpha-\frac{1}{2}u)}{A+B+C}$$

$$= \frac{Aa\cos(\alpha+\frac{1}{2}u)-Cb\cos(\alpha-\frac{1}{2}u)}{A+B+C}$$

$$z = \frac{Aa\cos(\alpha+\frac{1}{2}u)+(A+B)b\cos(\alpha-\frac{1}{2}u)}{A+B+C}$$

Præterea vero corpuſculorum celeritates ita ſe habebunt,
Secundum directionem Oo

$$\text{Cel. corp. A} = -\frac{(B+C)a\cos(\alpha+\frac{1}{2}u)+Cb\cos(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. corp. B} = \frac{Aa\cos(\alpha+\frac{1}{2}u)+Cb\cos(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. corp. C} = \frac{Aa\cos(\alpha+\frac{1}{2}u)-(A+C)b\cos.(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

At vero ſecundum directionem $O\omega$ erit

$$\text{Cel. Corp. A} = \frac{(B+C)a\sin(\alpha+\frac{1}{2}u)-Cb\sin(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. Corp. B} = -\frac{Aa\sin(\alpha+\frac{1}{2}u)-Cb\sin(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

$$\text{Cel. Corp. C} = -\frac{Aa\sin(\alpha+\frac{1}{2}u)+(A+B)b\sin(\alpha-\frac{1}{2}u)}{(A+B+C)\sqrt{g(m-2\cos u)}}$$

ſiquidem ponamus centrum gravitatis in puncto O quieſcere.

Exemplum.

24. Ponamus corpora extrema A & C inter ſe eſſe æqualia, & æqualiter a medio B remota, ita ut Fig. 5
ſit

ſit $C = A$ & $b = a$, eritque $m = \frac{2(A+B)}{A}$. Ponamus inſuper hæc tria corpora initio in directum fuiſſe poſita, ita ut ſumto $t = 0$ fiat quoque $u = 0$, atque ob x, y & $z = 0$ oportebit eſſe $\alpha = 90^{\circ}$. Ut igitur motus ad caſum II. componatur, celeritates corporum ſecundum directionem Oo evaneſcent, celeritates vero in directionibus ad axem normalibus ita erunt.

Corpus A habebit celeritatem $= \frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}}$ in directione $A\alpha$.

Corpus B habebit celeritatem $= \frac{2Aa\sqrt{A:B}}{(2A+B)\sqrt{2g}}$
in directione $B\beta$

Corpus C habebit celeritatem $= \frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}}$
in directione $C\gamma$

Cum igitur celeritates extremorum A & C ſint æquales, ponantur debitæ altitudini $= k$; erit $\frac{a\sqrt{AB}}{(2A+B)\sqrt{2g}} = \sqrt{k}$ & $\sqrt{2g} = \frac{a\sqrt{AB}}{(2A+B)\sqrt{k}}$; & celeritas medii in directione $B\beta$ erit $= \frac{2A\sqrt{k}}{B}$, & altitudo huic celeritati debita $=$ $\frac{4A^2k}{BB}$.

Nunc quæramus ſtatum horum corporum elapſo tempore t, quo

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quo generatur angulus u, ut ſit $t = \int \frac{du \sqrt{g(m - 2\cos u)}}{2}$

$$= \int \frac{du \sqrt{2g\left(\frac{A+B}{A} - \cos u\right)}}{2} =$$

$$\frac{a\sqrt{B}}{2(2A+B)\sqrt{k}} \int du \sqrt{(B + A - A\cos u)}$$ Fig. 6.

Inventoque hoc angulo $CB\gamma = u$, habebitur:

$p = -a\cos\tfrac{1}{2}u;\ q = 0;\ r = a\cos\tfrac{1}{2}u$

$$p = \frac{Ba\sin\tfrac{1}{2}u}{2A+B};\ y = -\frac{2Aa\sin\tfrac{1}{2}u}{2A+B};\quad z = \frac{Ba\sin\tfrac{1}{2}u}{2A+B}$$

Celeritates vero corpuſculorum ita ſe habebunt, ob

$$(A+B+C)\sqrt{g(m - 2\cos u)} = \frac{a\sqrt{B(B+A-A\cos u)}}{\sqrt{k}}$$

Secundum directionem Oo

$$\text{Corp. } A = \frac{(2A+B)\sin\tfrac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$$

$$\text{Corp. } B = 0$$

$$\text{Corp. } C = \frac{-(2A+B)\sin\tfrac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$$

Secundum directionem $O\omega$

$$\text{Corp. } A = \frac{B\cos\tfrac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$$

$$\text{Corp. } B = \frac{-2A\cos\tfrac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$$

$$\text{Corp. } C = \frac{B\cos\tfrac{1}{2}u}{\sqrt{B(B+A-A\cos u)}}\sqrt{k}$$

 Oc-

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Occupabunt ergo corpuſcula ABC elapſo tempore t, quo fila AB & CB ad angulum $CB\gamma = u$ inflectuntur, ejusmodi ſitum, quem figura indicat; eruntque ipſi anguli ad e & f, quibus fila ad axem $O\omega$ inclinantur, $BeO = BfO = \frac{1}{2}u$. Si ergo ponatur angulus $\frac{1}{2}u = \varphi$, ob $1 - \text{coſ}\,2\varphi = 2\,\text{ſin}\,\varphi^2$ erit $t = \frac{a\sqrt{B}}{(2A+B)\sqrt{k}} \int d\varphi \sqrt{(B + 2A\,\text{ſin}\,\varphi^2)}$, hincque ſimul celeritates ob $\sqrt{B(B+A-A\,\text{coſ}\,u)} = \sqrt{B(B+2A\,\text{ſin}\,\varphi^2)}$ ſimplicius exprimentur.

Scholion.

25. Poteſt hoc exemplum, quod Celeb. Daniel Bernoulli ad methodi meæ bonitatem explorandam mihi evolvendum propoſuit, etiam ſine ſubſidio ſolutionis generalis hic traditæ ex cognitis mechanicæ principiis reſolvi.
Fig. 6. Cum enim utrinque omnia ſint æqualia, perſpicuum eſt corpuſculum medium B alium motum habere non poſſe niſi ſecundum rectam $O\omega$, corpuſcula vero extrema A & C æqualiter ad rectam $O\omega$ vel accedere vel ab ea recedere oportere. Exqua circumſtantia per principium conſervationis virium vivarum ſingulorum corpuſculorum motus ſequenti modo determinari poterunt. Poſitis ut ante corpuſculorum A & C maſſis $= A$, corpuſculi B maſſa $= B$, & longitudine fili $AB = BC = a$, atque angulo $BeO = BfO = \varphi$, ſit celeritas corpuſculi B in directione $B\beta$ debita altitudini v; utriusque corpuſculi A & C ſit celeritas rotatoria circa B debita altitudini u; erit utriusque celeritas ſecundum directionem $O\omega$ debita altitudine $= u\,\text{coſ}\,\varphi^2$, & celeritas, qua utrumque directe ad $O\omega$ accedit debita altitudini

dini u ſinφ^2. Hinc erit celeritas, qua utrumque ab axe $O\omega$ recedit $= \cos\varphi\sqrt{u} - \sqrt{v}$, et quia centrum gravitatis in quiete manere ponitur, erit $B\sqrt{v} = 2A(\cos\varphi\sqrt{u} - \sqrt{v})$, ideoque $\sqrt{u} = \frac{(2A+B)\sqrt{v}}{2A\cos\varphi}$ ſeu $\sqrt{v} = \frac{2A\cos\varphi.\sqrt{u}}{2A+B.}$. Fig. 5.

Deinde cum initio utriusque corpuſculi A & C celerites ſecundum directionem $O\omega$ poſita ſit $=\sqrt{k}$, celeritas corpuſculi B in directione $B\beta = \frac{2A\sqrt{k}}{B}$, ſumma virium vivarum erat $= 2Ak + \frac{4AAk}{B} = \frac{2Ak}{B}(2A+B)$, quæ perpetuo eadem manere debebit. Præſenti autem caſu eſt corporis B vis viva $= Bv$, & corpus A, quia habet duplicem celeritatem alteram $= \cos\varphi\sqrt{u} - \sqrt{v}$, alteram vero $= \sin\varphi\sqrt{u}$, erit ejus vis viva $= A(\cos\varphi\sqrt{u} - \sqrt{v})^2 + Au\sin\varphi^2$ Fig. 6

$= A\left(\frac{B\cos\varphi\sqrt{u}}{2A+B}\right)^2 + Au\sin\varphi^2 = Au\left(\sin\varphi^2 + \frac{BB\cos\varphi^2}{(2A+B)^2}\right)$,

cui cum vis viva corporis C ſit æqualis, erit ſumma virum vivarum $= Bv + \frac{2Au}{(2A+B)^2}((2A+B)^2\sin\varphi^2 + B^2\cos\varphi^2)$

At eſt $Bv = \frac{4AAB\cos\varphi^2}{(2A+B)^2} = \frac{2Au}{(2A+B)^2}.\ 2AB\cos\varphi^2$;

unde tota vis viva erit $= \frac{2Au}{(2A+B)^2}((2A+B)^2\sin\varphi^2 + (2A+B)B\cos\varphi^2) = \frac{2Au}{2A+B}(2A\sin\varphi^2 + B)$, quæ cum æqualis eſſe debeat ſummæ virium vivarum initiali, erit

 2 A

$$\frac{2Au}{2A+B}(2A\,\text{ſin}\,\phi^2+B)=\frac{2Ak}{B}(2A+B),\text{atque}\;u(2A\,\text{ſin}\,\phi^2+B)$$
$$=\frac{k}{B}(2A+B)^2,\;\text{hincque}\;\sqrt{u}=\frac{(2A+B)\sqrt{k}}{\sqrt{B(B+2A\,\text{ſin}\,\phi^2)}}$$

Quoniam nunc celeritas rotatoria corpuſculi A eſt $\sqrt{u}$, hac tempuſculo infinite parvo dt arculum radio AB $= a$ deſcribet $= ad\phi$, eritque idcirco $\frac{ad\phi}{\sqrt{u}} = dt$, unde habebitur $dt =$
$\frac{ad\phi\sqrt{B(B+2A\,\text{ſin}\,\phi^2)}}{(2A+B)\sqrt{k}}$ & $t = \frac{a\sqrt{B}}{(2A+B)\sqrt{k}}\int d\phi\sqrt{(B+2A\,\text{ſin}\,\phi^2)}$
quæ eſt ea ipſa æquatio, quam ante invenimus; hocque adeo conſenſu methodi bonitas atque ſolutionis generalis veritas comprobatur.

Problema. IV.

Fig. 7. 26. *Sint nunc corpuſcula quotcunque* A, B, C, D, E, &c. *filo colligata, quæ ſi ſuper plano horizontali utcunque projiciantur, eorum motum inveſtigare.*

Solutio.

Ductis ex ſingulis corpuſculis ad axem fixum Oe perpendiculis vocentur:

	$Oa = p$;	$Ob = q$;	$Oc = r$;	$Od = s$	&c.
	$Aa = x$;	$Bb = y$;	$Cc = z$;	$Dd = v$	&c.
Filum	$AB = a$;	$BC = b$;	$CD = c$;	$DE = d$;	&c.
Ang.	$ABb = \zeta$;	$BCc = \eta$;	$CDd = \theta$;	$DEe = \iota$;	&c.

Ex

Ex quibus denominationibus deducuntur ſequentes æquationes.

$$q-p=a\,\text{ſin}\,\zeta;\quad r-q=b\,\text{ſin}\,\eta;\quad s-r=c\,\text{ſin}\,\theta;\quad \&c.$$
$$y-x=a\,\text{coſ}\,\zeta;\quad z-y=b\,\text{coſ}\,\eta;\quad v-z=c\,\text{coſ}\,\theta;\quad \&c.$$

Cum nunc ſpatiola hinc exprimi queant, quæ a ſingulis corpuſculis tempuſculo infinite parvo, quod ſit $=dt$, deſcribuntur, celeritates eorum tam ſecundum directionem axi O*o* parallelam, quam ad O*o* normalem ſequenti modo definientur.

Corpuſculi	celeritas in directione O*o* erit	celeritas in directione O*ω* erit
A	$\frac{dp}{dt}$	$\frac{dx}{dt}$
B	$\frac{dq}{dt}$	$\frac{dy}{dt}$
C	$\frac{dr}{dt}$	$\frac{dz}{dt}$
D	$\frac{ds}{dt}$	$\frac{dv}{dt}$
	&c.	&c.

Denotent jam litteræ A, B, C, D, &c. reſpective maſſas corpuſculorum, & quia eorum motus a tenſione filorum alteratur, ſit tenſio fili AB $=$ P; tenſio fili BC $=$ Q; tenſio fili CD $=$ R, &c. quibus poſitis.

Corpuſculum	ſollicitabitur in directione O*o* vi	ſollicitabitur in directione O*ω* vi
A	$P\,\text{ſin}\,\zeta$	$P\,\text{coſ}\,\zeta$
B	$Q\,\text{ſin}\,\eta - P\,\text{ſin}\,\zeta$	$Q\,\text{coſ}\,\eta - P\,\text{coſ}\,\zeta$
C	$R\,\text{ſin}\,\theta - Q\,\text{ſin}\,\eta$	$R\,\text{coſ}\,\theta - Q\,\text{coſ}\,\eta$
D	$S\,\text{ſin}\,\iota - R\,\text{ſin}\,\theta$	$S\,\text{coſ}\,\iota - R\,\text{coſ}\,\theta$
	&c.	&c.

 Ex

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Ex his ſollicitationibus ſequentes orientur accelerationes

$$\frac{2A\,ddp}{dt^2} = P\sin\zeta \qquad \frac{2A\,ddx}{dt^2} = P\cos\zeta$$

$$\frac{2B\,ddq}{dt^2} = Q\sin\eta - P\sin\zeta \qquad \frac{2B\,ddy}{dt^2} = Q\cos\eta - P\cos\zeta$$

$$\frac{2C\,ddr}{dt^2} = R\sin\theta - Q\sin\eta \qquad \frac{2C\,ddz}{dt^2} = R\cos\theta - Q\cos\eta$$

$$\frac{2D\,dds}{dt^2} = S\sin\iota - R\sin\theta \qquad \frac{2D\,ddv}{dt^2} = S\cos\iota - R\cos\theta$$

&c. &c.

Ex his æquationibus additis orientur iſtæ duæ æquationes

$$2A\,ddp + 2B\,ddq + 2C\,ddr + 2D\,dds + \&c. = 0$$

$$2A\,ddx + 2B\,ddy + 2C\,ddz + 2D\,ddv + \&c. = 0$$

quæ integratæ dant, diviſione per 2 inſtituta:

$$A\,dp + B\,dq + C\,dr + D\,ds + \&c. = \mathfrak{A}\,dt$$

$$A\,dx + B\,dy + C\,dz + D\,dv + \&c. = \mathfrak{B}\,dt$$

& integralibus denuo ſumtis;

$$Ap + Bq + Cr + Ds + \&c. = \mathfrak{A}t + a$$

$$Ax + By + Cz + Dv + \&c. = \mathfrak{B}t + \mathfrak{b}$$

quibus motus uniformis in directum centri gravitatis indicatur. Cum igitur ſit:

$$q = p + a\sin\zeta$$

$$r = p + a\sin\zeta + b\sin\eta$$

$$s = p + a\sin\zeta + b\sin\eta + c\sin\theta$$

&c.

y =

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$$y = x + a\cos\zeta$$
$$z = x + a\cos\zeta + b\cos\eta$$
$$v = x + a\cos\zeta + b\cos\eta + c\cos\theta \quad \&c.$$

Obtinebitur:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} - (B+C+D+\&c.)\,a\sin\zeta - (C+D+E+\&c.)\,b\sin\eta}{A+B+C+D+\&c} \frac{-(D+E+\&c.)\,c\sin\theta - \&c.}{A+B+C+D+\&c.}$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} - (B+C+D+\&c.)\,a\cos\zeta - (C+D+E+\&c.)\,b\cos\eta}{A+B+C+D+\&c.} \frac{-(D+E+\&c.)\,c\cos\theta - \&c.}{A+B+C+D+\&c.}$$

quæ formulæ in ſequentes transmutabuntur:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} + Aa\sin\zeta + (A+B)\,b\sin\eta + (A+B+C)\,c\sin\theta + \&c.}{A+B+C+E+\&c.}$$
$$-a\sin\zeta - b\sin\eta - c\sin\theta - \&c.$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} + Aa\cos\zeta + (A+B)\,b\cos\eta + (A+B+C)\,c\cos\iota + \&c.}{A+B+C+D+E+\&c.}$$
$$-a\cos\zeta - b\cos\eta - c\cos\iota - \&c.$$

quibus inventis ſimul litterarum $q, r, s,$ &c. & $y, z, v,$ &c. valores innoteſcent. Perducta eſt ergo quæſtio ad determinationem angulorum $\zeta, \eta, \theta, \iota,$ &c. qui ex duplicibus expreſſionibus tenſionum P, Q, R, &c. elicientur:

$$2P\,dt^2 = \frac{A\,ddp}{\sin\zeta} = \frac{A\,ddx}{\cos\zeta}$$
$$2Q\,dt^2 = \frac{A\,ddp + B\,ddq}{\sin\eta} = \frac{B\,ddx + B\,ddy}{\cos}$$

2R

$$\tfrac{1}{2} R\, dt^2 = \frac{A ddp + B ddq + C ddr}{\sin \iota} = \frac{A ddx + B ddy + C ddz}{\cos \iota}$$

&c.

ex his enim erir:

$$\frac{\sin \zeta}{\cos \zeta} = \frac{A ddp}{A ddx}$$

$$\frac{\sin \eta}{\cos \eta} = \frac{A ddp + B ddq}{A ddx + B ddy}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{A ddp + B ddq + C ddr}{A ddx + B ddy + C ddz}$$

&c.

Ponatur ſumma omnium corpuſculorum $A + B + D + \&c. = H$, & cum ſit:

$$p = \frac{\mathfrak{A}t + \mathfrak{a} - (H-A) a \sin \zeta - (H-A-B) b \sin \eta - (H-A-B-C) c \sin \theta - \&c.}{H}$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} - (H-A) a \cos \zeta - (H-A-B) b \cos \eta - (H-A-B-C) c \cos \theta - \&c.}{H}$$

ob:

$$Ap + Bq = (A + B)p + B a \sin \zeta$$

$$Ap + Bq + Cr = (A + B + C)p + (B + C) a \sin \zeta + C b \sin \eta$$

$$Ax + Bq + Cr + Ds = (A + B + C + D)p + (B + C + D) a \sin \zeta + (C + D) b \sin \theta \quad \&c.$$

$$Ax + By = (A + B)x + B a \cos \zeta$$

$$Ax + By + Cz = (A + B + C)x + (B + C) a \cos \zeta + C b \cos \eta$$

$$Ax + By + Cz + Dv = (A + B + C + D)x + (B + C + D) a \cos \zeta + (C + D) b \cos \eta + D c \cos \theta \quad \&c.$$

His

His igitur valoribus ſubſtitutis habebitur:

$$\frac{\sin\zeta}{\cos\zeta} = \frac{A(H-A)add\sin\zeta + A(H-A-B)bdd\sin\eta + A(H-A-B-C)cdd.\sin\theta + \&c.}{A(H-A)add.\cos\zeta + A(H-A-B)bdd\cos\eta + A(H-A-B-C)cdd\cos\theta + \&c.}$$

$$\frac{\sin\eta}{\cos\eta} = \frac{A(H-A-B)add.\sin\zeta + (A+B)(H-A-B)bdd.\sin\eta + (A+B)(H-A-B-C)cdd.\sin\theta + \&c.}{A(H-A-B)add.\cos\zeta + (A+B)(H-A-B)bdd.\cos\eta + (A+B)(H-A-B-C)cdd.\cos\theta + \&c.}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{A(H-A-B-C)add.\sin\zeta + (A+B)(H-A-B-C)bdd.\sin\eta + (A+B+C)(H-A-B-C)cdd.\sin\theta + \&c.}{A(H-A-B-C)cdd.\cos\zeta + (A+B)(H-A-B-C)bdd.\cos\eta + (A+B+C)(H-A-B-C)cdd.\cos\theta + \&c.}$$

&c.

Sicque prodibunt tot æquationes, quot habentur anguli ζ, η, θ, &c. ex quibus adeo ſinguli determinabuntur. Præterea vero ex formulis pro ſollicitationibus momentaneis inventis eruentur ſimili modo, quo ſupra fuimus uſi:

$$2A(dpddp + dxddx) + 2B(dqddq + dyddy) + 2C(drddr + dzddz) + \&c. = 0$$

quæ integrata dabit:

$$A(dp^2 + dx^2) + B(dq^2 + dy^2) + C(dr^2 + dz^2) + \&c. = \mathfrak{S}\,dt^2$$

qua conſervatio virium vivarum continetur. Solutionem ergo illæ æquationes differentio-differentiales complectentur, quæ ſi integrationem admitterent, uti caſu trium corpuſculorum uſu venit, problema perfecte eſſet ſolutum. Suf-

ficiet ergo hujus problematis ſolutionem ad reſolutionem æquationum analyticarum perduxiſſe. Q.E.J.

Coroll. I.

27. Ex æquationibus, quibus anguli ζ, η, θ, &c. definiuntur, ſi fractiones tollantur, atque lemma hoc in ſubſidium vocetur $\cos m\, dd.\sin n - \sin m\, dd.\cos n = ddn \cos(m-n) + dn^2 \sin(m-n)$, orientur ſequentes:

$$0 = A(H-A)\,a\,dd\zeta + A(H-A-B)\,b\,(dd\eta \cos(\zeta-\eta) + d\eta^2 \sin(\zeta-\eta)) + A(H-A-B-C)\,c\,(dd\theta \cos(\zeta-\theta) + d\theta^2 \sin(\zeta-\theta)) + A(H-A-B-C-D)\,d\,(dd\iota \cos(\zeta-\iota) + d\iota^2 \sin(\zeta-\iota)) + \&c.$$

$$0 = A(H-A-B)\,a\,(dd\zeta \cos(\zeta-\eta) - d\zeta^2 \sin(\zeta-\eta)) + (A+B)(H-A-B)\,b\,dd\eta + (A+B)(H-A-B-C)\,c\,(dd\theta \cos(\eta-\theta) + d\theta^2 \sin(\eta-\theta)) + (A+B)(H-A-B-C-D)\,d\,(dd\iota \cos(\eta-\iota) + d\iota^2 \sin(\eta-\iota)) + \&c.$$

$$0 = A(H-A-B-C)\,a\,(dd\zeta \cos(\zeta-\theta) - d\zeta^2 \sin(\zeta-\theta)) + (A+B)(H-A-B-C)\,b\,(dd\eta \cos(\eta-\theta) - d\eta^2 \sin(\eta-\theta)) + (A+B+C)(H-A-B-C)\,c\,dd\theta + (A+B+C)(H-A-B-C-D)\,d\,(dd\iota \cos(\theta-\iota) + d\iota^2 \sin(\theta-\iota)) + \&c.$$

o =

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$$0 = A(H-A-C-D)\,a\,(dd\zeta\,\text{coſ}(\zeta-\iota) - d\zeta^2\,\text{ſin}(\zeta-\iota)) + (A+B)$$
$$(H-A-B-C-D)\,b\,(dd\eta\,\text{coſ}(\eta-\iota) - d\eta^2\,\text{ſin}(\eta-\iota))$$
$$+ (A+B+C)(H-A-B-C-D)\,c\,(dd\theta\,\text{coſ}(\theta-\iota) - d\theta^2\,\text{ſin}$$
$$(\theta-\iota))) + (A+B+C+D)(H-A-B-C-D)\,d\,dd\iota + \&c.$$
&c.

Coroll. 2.

28. Cum deinde ſit $\int(ddn\,\text{coſ}(m-n) + dn^2\,\text{ſin}(m-n)) = dn\,\text{coſ}(m-n) + \int dm\,dn\,\text{ſin}(m-n)$, erit æquationibus illis hoc modo integratis:

$$0 = A(H-A)\,a\,d\zeta$$
$$+ A(H-A-B)\,b\,d\eta\,\text{coſ}(\zeta-\eta) + A(H-A-B)\,b\int d\zeta\,d\eta\,\text{ſin}(\zeta-\eta)$$
$$+ A(H-A-B-C)\,c\,d\theta\,\text{coſ}(\zeta-\theta) + A(H-A-B-C)\,c\int d\zeta\,d\theta\,\text{ſin}(\zeta-\theta)$$
$$+ A(H-A-B-C-D)\,d\,d\iota\,\text{coſ}(\zeta-\iota) + A(H-A-B-C-D)\,d\int d\zeta\,d\iota\,\text{ſin}$$
$$(\zeta-\iota) \qquad \&c.$$

$$0 = A(H-A-B)\,a\,d\zeta\,\text{coſ}(\zeta-\eta) - A(H-A-B)\,a\int d\zeta\,d\eta\,\text{ſin}(\zeta-\eta)$$
$$+ (A+B)(H-A-B)\,b\,d\eta$$
$$+ (A+B)(H-A-B-C)\,c\,d\theta\,\text{coſ}(\eta-\theta) + (A+B)(H-A-B-C)$$
$$c\int d\eta\,d\theta\,\text{ſin}(\eta-\theta)$$
$$+ (A+B)(H-A-B-C-D)\,d\,d\iota\,\text{coſ}(\eta-\iota) + (A+B)$$
$$(H-A-B-C-D)\,d\int d\eta\,d\iota\,\text{ſin}(\eta-\iota) \qquad \&c.$$

$$0 = A(H-A-B-C)\,a\,d\zeta\,\text{coſ}(\zeta-\theta) - A(H-A-B-C)\,a\int d\eta\,d\theta\,\text{ſin}(\zeta-\theta)$$
$$+ (A+B)(H-A-B-C)\,b\,d\eta\,\text{coſ}(\eta-\theta) - (A+B)(H-A-B-C)$$
$$b\int d\eta\,d\theta\,\text{ſin}(\zeta-\theta)$$
$$+ (A+B+C)(H-A-B-C)\,c\,d\theta$$
$$+ (A+B+C)(H-A-B-C-D)\,d\,d\iota\,\text{coſ}(\eta-\iota) + (A+B+C)$$
$$(H-A-B-C-D)\,d\int d\theta\,d\iota\,\text{ſin}(\theta-\iota) \qquad \&c.$$

0 =

$$0 = A(H-A-B-C-D)\,ad\zeta\cos(\zeta-\iota) - A(H-A-B-C-D)\,a\int d\zeta\, d\iota \sin(\zeta-\iota)$$
$$+(A+B)(H-A-B-C-D)\,bd\zeta\cos(\eta-\iota) - (A+B)(H-A-B-C-D)\,b\int d\eta\, d\iota\sin(\eta-\iota)$$
$$+(A+B+C)(H-A-B-D)\,cd\theta\cos(\theta-\iota) - (A+B+C)(H-A-B-C-D\,c\int d\theta\, d\iota\sin(\theta-\iota)$$
$$+(A+B+C+D)(H-A-B-D)\,dd\iota$$
&c.

Coroll. 3.

29. Si harum æquationum prima mutiplicetur per a ſecundum per b, tertia per c, quarta per d, &c. omnesque invicem addantur termini integrales deſtruentur, prodibitque ſequens æquatio integralis.

$$\mathfrak{S}dt = A(H-A)\,aad\zeta + (A+B)(H-A-B)\,bbd\eta + (A+B+C)(H-A-B-C)\,ccd\theta + \&c.$$
$$+A(H-A-B)\,ab(d\zeta+d\eta)\cos(\zeta-\eta)$$
$$+A(H-A-B-C)\,ac(d\zeta+d\theta)\cos(\zeta-\theta)$$
$$+A(H-A-B-C-D)\,ad(d\zeta+d\iota)\cos(\zeta-\iota)\quad \&c.$$
$$+(A+B)(H-A-B-C)\,bc(d\eta+d\theta)\cos(\eta-\theta)$$
$$+(A+B)(H-A-B-C-D)\,bd(d\eta+d\iota)\cos(\eta-\iota)\quad \&c.$$
$$+(A+B)(H-A-B-C-D)\,bd(d\theta+d\iota)\cos(\theta-\iota)\quad \&c.$$

Eadem vero æquatio jam continetur in ea, qua conſervationem virium vivarum ſumus complexi.

Scholion.

30. Quo igitur poſitio corpuſculorum horum A, B, C, D, &c. ad quodvis tempus definiri, atque adeo eorum motus

motus perfecte cognoſci poſſit, æquationes has differentio-differentiales inventas reſolvi atque integrari oportet, quemadmodum in caſu trium corporum fieri licuit. At vero hic multitudo variabilium ſimilem tractationem impedit, neque quemadmodum hinc commoda conſtructio obtineri poſſit, perſpicitur. Quæ difficultas, quo clarius ob oculos ponatur, quatuor tantum corpora contemplemur.

Exemplum.

31. Sint quatuor corpuſcula filis inter ſe connexa, ſintque tam ipſa corpuſcula A, B, C, D, quam fila a, b, c, inter ſe æqualia, erit $H = 4A$, atque anguli ζ, η, θ ex tribus ſequentibus æquationibus inveſtigari debebunt.

$$0 = 3\,dd\zeta + 2\,dd\eta \cos(\zeta - \eta) + 2\,d\eta^2 \sin(\zeta - \eta) + dd\theta \cos(\zeta - \theta) + d\theta^2 \sin(\zeta - \theta)$$

$$0 = 2\,dd\zeta \cos(\zeta - \eta) - 2\,d\zeta^2 \sin(\zeta - \eta) + 4\,dd\eta + 2\,dd\theta \cos(\eta - \theta) + 2\,d\theta^2 \sin(\eta - \theta)$$

$$0 = dd\zeta \cos(\zeta - \theta) - d\zeta^2 \sin(\zeta - \theta) + 2\,dd\eta \cos(\eta - \theta) - 2\,d\eta^2 \sin(\eta - \theta) + 3\,dd\theta$$

ex quibus ſequens æquatio integralis (29) elicitur

$$\mathfrak{S}dt = 3\,d\zeta + 4\,d\eta + 3\,d\theta + 2\,(d\zeta + d\eta)\cos(\zeta - \eta) + (d\zeta + d\theta)\cos(\zeta - \theta) + 2\,(d\eta + d\theta)\cos(\eta - \theta)$$

Unde ſi valores angulorum ζ, η, & θ per t exprimi poſſent, foret:

 $p =$

$$p = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} - \frac{3a}{4}\sin\zeta - \frac{2a}{4}\sin\eta - \frac{a}{4}\sin\theta$$

$$p = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta - \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$r = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta - \tfrac{1}{4}a\sin\theta$$

$$s = \frac{\mathfrak{A}t + \mathfrak{a}}{4A} + \tfrac{1}{4}a\sin\zeta + \tfrac{2}{4}a\sin\eta + \tfrac{3}{4}a\sin\theta$$

atque

$$x = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} - \tfrac{3}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$y = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta - \tfrac{2}{4}a\cos\eta - a\cos\theta$$

$$z = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta - \tfrac{1}{4}a\cos\theta$$

$$v = \frac{\mathfrak{B}t + \mathfrak{b}}{4A} + \tfrac{1}{4}a\cos\zeta + \tfrac{2}{4}a\cos\eta + \tfrac{3}{4}a\cos\theta$$

Problema. V.

Fig. 8. 32. *Augeatur nunc numerus corpuſculorum in infinitum, filorum autem longitudines evaneſcant, ita ut hoc modo funis perfecte flexibilis formetur, cujus, ſi ſuper plano horizontali utcunque projiciatur, motus & ſitus ad quodvis tempus aſſignari debet.*

Solutio.

Pervenerit iſte funis elapſo tempore t in ſitum A MG, ex cujus ſingulis punctis M perpendicula ad axem Oo demiſſa con-

concipiantur. Vocetur abſciſſa quæcunque $OP = X$, applicata $PM = Y$, & longitudo portionis funis $AM = S$, ejus vero maſſa exprimatur per functionem quamcunque Σ ipſius S, ſitque præterea angulus $AMP = \Phi$, ita ut ſit $dX = dS \sin \Phi$ & $dY = dS \cos \Phi$. Ponatur ut ante $Oa = p$ & $Aa = x$, quia invenimus

$$p = \frac{\mathfrak{A}t + \mathfrak{a} + Aa \sin \zeta + (A+B) b \sin \eta + (A+B+C) c \sin \theta + \&c.}{A+B+C+E+\&c.}$$

$$x = \frac{\mathfrak{B}t + \mathfrak{b} + Aa \cos \zeta + (A+B) b \cos \eta + (A+B+C) c \cos \theta + \&c.}{A+B+C+D+\&c.}$$

ſi maſſam totius funis ponamus $= H$, his formulis ad caſum præſentem translatis habebimus

$$Oa = \frac{\mathfrak{A}t + \mathfrak{a} + \int \Sigma dS \sin \Phi}{H} - \int dS \sin \Phi$$

$$Aa = \frac{\mathfrak{B}t + \mathfrak{b} + \int \Sigma dS \cos \Phi}{H} - \int dS \cos \Phi$$

his integralibus per totam funis longitudinem AMG extenſis. Erit autem $\int dS \sin \Phi = ag$ & $\int dS \cos \Phi = Gg - Aa$. Deinde cum ſupra invenerimus: $Ap + Bq + Cr + Ds + \&c. = (A+B+C+D+\&c.)(p + a \sin \zeta + b \sin \eta + c \sin \theta + \&c.) - Aa \sin \zeta - (A+B) b \sin \eta - (A+B+C) c \sin \theta - \&c.$ erit hæc expreſſio ad præſentem caſum translata pro arcu $AM = \Sigma(Oa + \int dS \sin \Phi) - \int \Sigma dS \sin \Phi = Oa.\Sigma + \int d\Sigma \int dS \sin \Phi$. Simili modo & altera expreſſio $Ax + By + Cz + Dv + \&c.$ noſtro caſu tranſit in hanc $Aa.\Sigma + \int d\Sigma \int dS \cos \Phi$; unde erit $\frac{\sin \Phi}{\cos \Phi} = \frac{dd.(Oa.\Sigma + \int d\Sigma \int dS \sin \Phi)}{dd.(Aa.\Sigma + \int d\Sigma \int dS \cos \Phi)}$, quæ differentialia

ſecun-

ſecundi gradus ex variabilitate temporis ſola ſunt deſumenda, ita ut S & Σ tanquam conſtantia tractentur.

Quoniam ergo angulus Φ cum tempore t variatur, etiamſi arcus S idem maneat, quantitas Φ erit functio duarum variabilium S & t. Ponatur propterea $d\Phi = M\,dS + N\,dt$ eritque poſito S conſtante & ſolo t variabilis.

$$d.Oa = \frac{\mathfrak{A}\,dt + dt\int \Sigma N\,dS \cos\Phi}{H} - dt\int N\,dS \cos\phi$$

$$d.Aa = \frac{\mathfrak{B}\,dt - dt\int \Sigma N\,dS \sin\phi}{H} + dt\int N\,dS \sin\phi$$

Quia vero N eſt porro functio ipſarum S & t ponatur $dN = P\,dS + Q\,dt$, eritque poſito ſolo t variabili, & dt conſtante

$$dd.Oa = -\frac{dt^2\int \Sigma N^2\,dS \sin\phi + dt^2\int \Sigma Q\,dS \cos\phi}{H}$$

$$+ dt^2\int N^2\,dS \sin\phi - dt^2\int Q\,dS \cos\phi$$

$$dd.Aa = -\frac{dt^2\int \Sigma N^2\,dS \cos\phi - dt^2\int \Sigma Q\,dS \sin\phi}{H}$$

$$+ dt^2\int N^2\,dS \cos\phi + dt^2\int Q\,dS \sin\phi$$

quæ ſingula integralia ad totam curvam erunt extendenda; ita ut ea abeant in functiones ipſius t tantum; hancobrem ſit $dd.Oa = E\,dt^2$ & $dd\,Aa. = F\,dt^2$, erunt E & F functiones ipſius t tantum. Deinde quæ pertinent ad ſolum arcum indeſinitum AM, erunt differentialia, quæ ex ſola variabilitate ipſius t oriuntur:

d.ſd

$$d.\int d\Sigma \int dS \sin\phi = dt \int d\Sigma \int N\, dS \cos\phi$$

$$dd.\int d\Sigma \int dS \sin\phi = -dt^2 \int d\Sigma \int N^2\, dS \sin\phi + dt^2 \int d\Sigma \int Q\, dS \cos\phi$$

$$d.\int d\Sigma \int dS \cos\phi = -dt \int d\Sigma \int N\, dS \sin\phi$$

$$dd.\int d\Sigma \int dS \cos\phi = -dt^2 \int d\Sigma \int N^2\, dS \cos\phi - dt^2 \int d\Sigma \int Q\, dS \sin\phi$$

Ex his ergo obtinebitur ſequens æquatio;

$$\frac{\sin\phi}{\cos\phi} = \frac{E\Sigma - \int d\Sigma \int N^2\, dS \sin\phi + \int d\Sigma \int Q\, dS \cos\phi}{F\Sigma - \int d\Sigma \int N^2\, dS \cos\phi - \int d\Sigma \int Q\, dS \sin\phi}$$

Quo autem naturam hujus curvæ ejusque motus commodius exprimamus, ponamus:

$$Oa.\,\Sigma + \int d\Sigma \int dS \sin\phi = T$$

$$Aa.\,\Sigma + \int d\Sigma \int dS \cos\phi = V$$

ſitque nobis δ character differentialium, quæ ex variabilitate ipſius t naſcuntur, manente d charactere differentialium tantum, quæ ex ſola variabilitate ipſius S ſeu Σ ortum trahunt.

Erit ergo $\dfrac{\sin\phi}{\cos\phi} = \dfrac{\delta\delta T}{\delta\delta V}$: tum vero differentiando formulas antecedentes ponendo tantum Σ vel S variabili, ob Oa & Aa conſtantes erit

$$Oa.\,d\Sigma + d\Sigma \int dS \sin\phi = dT$$

$$Aa.\,d\Sigma + d\Sigma \int dS \cos\phi = dV$$

& ſtatuendo $d\Sigma$ conſtante, ita ut S tanquam functio ipſius Σ ſpectetur, ſi denuo differentialia ſumantur, erit

$$d\Sigma\, dS \sin\phi = ddT$$

$$d\Sigma\, dS \cos\phi = ddV$$

ideoque $\frac{\text{ſin}\,\phi}{\text{coſ}\,\phi} = \frac{ddT}{ddV}$, unde obtinetur iſta æquatio

$$\frac{\delta\delta T}{\delta\delta V} = \frac{ddT}{ddV}$$

Præterea vero cum ſit $\text{ſin}\,\phi = \frac{dd\,T}{dz\,dS}$, & $\text{coſ}\,\phi = \frac{dd\,V}{dz\,dS}$

debebit eſſe $ddT^2 + ddV^2 = dz^2\,dS^2$. Quæſtio ergo huc redit ut inveſtigentur duæ hujusmodi functiones ipſarum z & t quæ ſint T & V, ita ut earum differentialia ſecunda poſito ſolo z variabili eandem inter ſe teneant rationem, quam earundem functionum differentialia ſecunda, ſi ſolum t ponatur variabile: præterea vero debet eſſe $ddT^2 + ddV^2 = dS^2\,dz^2$: quibus inventis erit

$$\text{ſin}\,\phi = \frac{dd\,T}{dS\,dz} \quad \& \ \text{coſ}\,\phi = \frac{dd\,V}{dS\,dz}.$$

Hincque porro coordinatæ X & Y reperientur. Vel cum ſit $dX = dS\,\text{ſin}\,\phi$ & $dY = dS\,\text{coſ}\,\phi$, erit $ddT = dz\,dX$ & $ddV = dz\,dY$, unde ſit $T = \int X\,dz$ & $V = \int Y\,dz$. Quare X & Y ita comparatæ eſſe debent, ut ſit $\frac{\delta\delta T}{\delta\delta V} = \frac{dX}{dY}$ vel $\frac{\int dz\,\delta\delta X}{\int dz\,\delta\delta Y} = \frac{dX}{dY}$. Reducta ergo eſt ſolutio hujus problematis mechanici ad problema analyticum; in quo acquieſcere oportet. Q. E. J.

Scholion. I.

33. Quo hæc clarius explicentur, ponamus funem ubique eſſe æque craſſum, ita ut ſit maſſa z proportionalis longi-

longitudini S. Sit portio $AM = S$, angulus $AMP = \varphi$, & longitudo tota $AMG = h$, eritque elapſo tempore t

$$Oa = \mathfrak{A}t + \alpha + \frac{\int s\,ds \sin\varphi - h\int ds \sin\varphi}{h}$$

ubi integralia $\int ds \sin\varphi$ ita debent capi, ut evaneſcant poſito $s = 0$, tum vero ſtatui oportet $s = h$, ſicque Oa exprimetur per functionem ipſius t. Simili modo erit

$$Aa = \mathfrak{B}t + \mathfrak{b} + \frac{\int s\,ds \cos\varphi - h\int ds \cos\varphi}{h}$$

ubi integralia $\int ds \cos\varphi$ & $\int sds \cos\varphi$ pariter ita accipi debent ut evaneſcant poſito $s = 0$, quo facto ubique faciendum eſt $s = h$.

Statuatur brevitatis ergo:	fiatque caſu $s = h$
$\int ds \sin\varphi = P$	$P = A$
$\int ds \cos\varphi = Q$	$Q = B$
$\int sds \sin\varphi = R$	$R = C$
$\int sds \cos\varphi = S$	$S = D$

erit $Oa = \mathfrak{A}t + \alpha + \dfrac{C - Ah}{h}$;

$$Aa = \mathfrak{B}t + \mathfrak{b} + \frac{D - Bh}{h}$$

Deinde ſuperior expreſſio generalis $\int d z \int dS \sin\varphi$ abit hic in $Ps -$, & $\int dz \int dS \cos\varphi$ in $Qs - S$; ideoque ſi δ ſumatur pro charactere differentiationis, ſi t tantum variabile ſpectetur, erit

$$\frac{\sin\varphi}{\cos\varphi} = \frac{\delta\delta(Cs - Ahs + Phs - Rh)}{\delta\delta(Ds - Bbs + Qhs - Sh)}$$

 ubi

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ubi omiſimus $\mathfrak{A}t + \mathfrak{a}$ & $\mathfrak{B}t + \mathfrak{b}$, quia horum differentialia ſecunda evaneſcunt. Quanquam autem hinc idoneæ functiones ipſarum t & s pro angulo φ adhibendæ inveniri vix poſſunt, tamen ope harum valores, quos quis forte pro φ exhibuerit, facile explorari poſſunt, utrum problemati ſatisfaciant necne.

Scholion. 2.

34. Quamvis autem hoc problema ſit difficillimum, ſi in genere conſideretur, tamen unus extat caſus ſpecialis, quo ſolutu ſit facillimum. Hic locum habet, ſi angulus φ exprimatur per functionem ipſius s tantum, ita ut in eam tempus t non ingrediatur. Quia enim tum formulæ $Cs - Ahs + Phs - Rh$ & $Ds - Bhs + Qhs - Sh$ a ſola variabili s pendent, earum differentialia, quæ prodeunt, ſi ſolum t variabile ponatur, evaneſcent, ſicque æquationi ultimæ ſatisfit. Hoc igitur caſu funis inſtar corporis rigidi motu ſibi parallelo feretur, ita ut ſingulæ ejus partes perpetuo ad axem Oo eandem inclinationem conſervent, & ſingulorum punctorum M celeritates, tam ſecundum directionem axis Oo, quam axis ad eum normalis $O\omega$ inter ſe erunt æquales. Quare ſi funi initio hujusmodi motus fuerit impreſſus, ut primo ſaltem momento ſingula elementa ad axem Oo eandem inclinationem retineant, tum eodem motu perpetuo promoveri perget.

Coroll. 1.

35. Si ponatur $Cs - Ahs + Phs - Rh = T$ & $Ds - Bhs + Qhs - Sh = \Theta$, erunt T & Θ ejusmodi functiones, quæ evaneſcunt tam ſi ponatur $s = 0$, quam ſi fiat

fiat $s = h$. Ita autem præterea iſtæ functiones T & Θ debent eſſe comparatæ, ut ſit

$$dd\mathrm{T} = hds^2 \sin\varphi \quad dd\Theta = hds^2 \cos\varphi.$$

in quibus differentiationibus ſolum s poſitum eſt variabile & ds conſtans. Sin autem ſolum t variabile ſtatuatur, debet eſſe

$$\delta\delta\mathrm{T} = \mathrm{V}dt^2 \sin\varphi \quad \& \ \delta\delta\Theta = \mathrm{V}dt^2 \cos\varphi$$

exiſtente V functione quacunque.

Coroll. 2.

36. Si igitur hujusmodi functiones T & Θ inveniri poſſent, haberetur ſolutio particularis problematis; ſin autem omnes omnino functiones his proprietatibus gaudentes in formulis generalibus comprehendi poſſent, tum haberetur ſolutio problematis generalis abſoluta, qualis deſideratur.

Coroll. 3.

37. Sint M & N & φ functiones ipſarum s & t, & ponatur:

$$\mathrm{T} = \mathrm{M}\sin\varphi + \mathrm{N}\cos\varphi; \quad \Theta = \mathrm{M}\cos\varphi - \mathrm{N}\sin\varphi$$

atque problemati ſatisfiet, ſi primum T & Θ evaneſcant tam caſu $s = 0$ quam caſu $s = h$; quod fit, ſi his caſibus & M & N fiant $= 0$. Deinde vero requiritur ut ſit

$$hds^2 = dd\mathrm{M} - \mathrm{M}d\varphi^2 - 2d\mathrm{N}d\varphi - \mathrm{N}dd\varphi$$

Præterea vero debet eſſe

$$dd\mathrm{N} - \mathrm{N}d\varphi^2 + \mathrm{M}dd\varphi + 2d\mathrm{M}d\varphi = 0$$
$$\delta\delta\mathrm{N} - \mathrm{N}\delta\varphi^2 + \mathrm{M}\delta\delta\varphi + 2\delta\mathrm{M}\delta\varphi = 0$$

in quarum æquationum prima & ſecunda t & ds ſunt conſtantia, in tertia vero s & dt conſtantia ſunt poſita.

Coroll. 4.

38. Ex æquationibus prima & ſecunda eliminando terminos continentes $d\varphi^2$ obtinebitur hæc:

$$(M^2+N^2)dd\varphi + 2d\varphi(MdM+NdN) = NddM$$
$$- MddN - Nhds^2$$

ex cujus integratione eruitur

$$(M^2+N^2)\,d\varphi = NdM - MdM - hds\int Nds$$

hincque porro $\varphi = A\ \mathrm{tag}\ \frac{M}{N} - h\int\frac{ds\int Nds}{M^2+N^2}$. Qui valor ſi in alterutra æquatione ſubſtituatur ponendo $\int Nds = K$ prodibit

$$MddM+NddN = hMds^2 + \frac{hhK^2ds}{M^2+N^2} - \frac{(NdM-MdN)^2}{M^2+N^2}$$

Addatur utrinque dM^2+dN^2 & ponatur $M^2+N^2=v^2$ erit

$$v\,ddv = \frac{hhK^2ds^2}{v^2} + hMds^2 = \frac{hhK^2ds^2}{v^2} +$$
$$hds^2\sqrt{(v^2-N^2)}$$

ex qua ſi definiatur v, ob K datum, per N, invenietur valor idoneus pro M ſubſtituendus. Tum vero N ex tertia æquatione determinari debet.

Co-

Coroll. 5.

39. Si ſit $N = 0$, æquatio ſecunda ſtatim dat $M^2 d\varphi = E ds$ & $d\varphi = \frac{E ds}{M^2}$, qui valor in prima $h ds^2 = ddM - M ds^2$ ſubſtitutus dabit, $h ds^2 = ddM - \frac{E^2 ds^2}{M^3}$. Statuatur $ds = v dM$, ut, ob ds conſtans ſit $ddM = \frac{-dv dM}{v}$, erit ſubſtitutione facta;

$$h v^2 dM = \frac{-dv}{v} - \frac{E^2 v^2 dM}{M^3} \quad \text{ſeu}$$

$$h dM = \frac{-dv}{v^3} - \frac{E^2 dM}{M^3}$$

cujus integrale eſt:

$$hM = \frac{1}{2v^2} + \frac{E^2}{2M^2} - F \quad \text{ſeu}$$

$$2hv^2M^3 = M^2 + E^2v^2 - 2Fv^2M^2 \text{ unde}$$

$$v = \frac{\pm M}{\sqrt{(2hM^3 + 2FM^2 - E^2)}}$$

$$\& \quad s = \int \frac{\pm M \ M}{\sqrt{(2hM + 2FM^2 - E^2)}}$$

Quoniam vero M debet evaneſcere poſito tam $s = 0$ quam $s = h$, fiat $\int \frac{\pm M dM}{\sqrt{(2hM^3 + 2FM^2 - E^2)}} = \pm G$ poſito $M = 0$, atque G determinabitur per E, F & conſtantes.

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ſtantes. Fiat ergo $G = \frac{1}{2} h$, & conditiones requiſitæ implebuntur, ſi ponatur

$$s = \frac{1}{2} h \pm \int \frac{M dM}{\sqrt{(2hM^3 + 2FM^2 - E^2)}}$$

unde erit $d\phi = \frac{\pm E dM}{M\sqrt{(2hM^3 + 2FM^2 - E^2)}}$

ubi E & F erunt quantitates tum ex conſtantibus tum ex t compoſitæ. Præterea vero ſi fieri poteſt, ita debent eſſe comparatæ, ut etiam tertiæ æquationi $M\partial\partial\phi + 2\partial M\, d\phi = 0$ ſeu $MM\partial\phi = H dt$ ſatisfiat, quod fiet ſi $d\phi + \partial\phi = -\frac{E ds + H dt}{MM}$ fuerit integrabile: ejus enim integrale verum dabit angulum ϕ. Ubi notandum eſt, H deſignare functionem quamcunque ipſius s non involventem t, uti E eſt functio ipſius t non continens s. Hancobrem $\frac{E ds + H dt}{MM}$ erit integrabile ſi fuerit $MM = EH$ in functionem quampiam quantitatis $\int\frac{ds}{H} + \int\frac{dt}{E}$, ſeu ſi ſit $\frac{MM}{EH}$ functio hujus quantitatis $\int\frac{ds}{H} + \int\frac{dt}{E}$.

Problema. VI.

Fig. 9. 40. *Conſtet corpus ABC duobus articulis AB & BC in B flexura invicem conjunctis, ita ut ambo circa B liberrime circumagi queant; quæriturque motus, quo hoc corpus ſuper plano, horizontali politiſſimo ſit progreſſurum, poſtquam ipſi ſemel motus quicunque fuerit impreſſus.*

So-

Solutio.

Sumtis pro lubitu in plano horizontali duobus axibus orthogonalibus $O o$ & $O \omega$ ſeſe decuſſantibus, ad quos quovis momento poſitio corporis referatur, pervenerit elapſo tempore quocunque t, corpus in ſitum ABC, ex cujus punctis A, B, C ad axem $O o$ demittantur perpendicula $A a$, $B b$, $C c$. Sit porro K centrum gravitatis articuli AB, & L centrum gravitatis articuli BC, ex quibus pariter ad axem $O o$ normales ducantur KP & LQ, vocenturque:

$$AK = \alpha;\ BK = b;\ BL = \beta;\ LC = c:$$
$$OP = p;\ PK = x;\ OQ = q;\ QL = y:$$
$$\text{Ang. } AKP = \zeta;\quad \& \text{ ang. } BLQ = \eta.$$

ex quibus erit

$$q - p = b \operatorname{ſin} \zeta + \beta \operatorname{ſin} \eta \quad \& \ y - x = b \operatorname{coſ} \zeta + \beta \operatorname{coſ} \eta.$$

Motus autem utriusque articuli, conſtat ex motu centri gravitatis, quem reſolvamus ſecundum directiones amborum axium $O o$ & $O \omega$, & ex motu rotatorio, quo uterque articulus circa ſuum gravitatis centrum gyrabitur. Erit ergo

Celeritas puncti	ſecundum directionem $O o$	ſecundum directionem $O \omega$
K	$= \frac{dp}{dt}$	$= \frac{dx}{dt}$
L	$= \frac{dq}{dt}$	$= \frac{dy}{dt}$

Celeritas rotatoria articuli AB circa centrum gravitatis K erit $= \frac{d\zeta}{dt}$ in diſtantia $= 1$.

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Celeritas rotatoria articuli BC circa centrum gravitatis L

erit $= \frac{d\eta}{dt}$ in diſtantia $= 1$.

Nunc ad motum determinandum ſit:

maſſa articuli AB $=$ K: articuli BC $=$ L

momentum inertiæ articuli AB $= K k^2$; articuli BC $= L l^2$

Momenta hæc inertiæ reſpectu utriusque articuli centri gravitatis ſumi ponuntur, eſtque momentum inertiæ aggregatum omnium corporis particularum per quadrata diſtantiarum ſuarum ab axe, circa quem corpus mobile concipitur, multiplicatarum. His poſitis ſi ambo articuli a ſe invicem eſſent diſſoluti, uterque eundem motum tam progreſſivum centri gravitatis quam rotatorium circa axem verticalem per centrum gravitatis transeuntem perpetuo conſervaret. Quoniam autem junctura in B ſunt colligati, ambo iſti motus ſe mutuo continuo perturbabunt, hæque perturbationes provenient a vi, quam junctura in B ſuſtinet. Quæ vis cum ſit incognita, ponamus ab ea articulum AB urgeri duabus viribus, altera in directione Bk quæ ſit $=$ B, altera in directione B$\varkappa$ quæ ſit $= \mathfrak{B}$; atque alter articulus BC iisdem viribus, at in directionibus oppoſitis urgebitur, ſcilicet in directione Bl vi $=$ B & in directione Bλ vi $= \mathfrak{B}$. His ergo viribus primum motus centri gravitatis afficietur, ac primo quidem vis B$\varkappa = \mathfrak{B}$ accelerabit motum centri gravitatis K ſecundum Oo, & vis B$k =$ B motum in directione Oω; contra vero vis B$\lambda = \mathfrak{B}$ retardabit motum centri gravitatis L ſecundum Oo, & vis B$l =$ B motum in directione Oω. Hinc ex legibus ſollicitationum erit:

$$2\,K\,ddp = \mathfrak{B}dt^2; \quad 2\,K\,ddx = B\,dt^2$$

2 L

$$2\,L\,ddq = -\mathfrak{B}dt^2\,;\quad 2\,L\,ddy = -B\,dt^2$$

Porro momentum vis $B\kappa = \mathfrak{B}$ reſpectu centri gravitatis K eſt $= \mathfrak{B}b\cos\zeta$, eoque motus rotatorius articuli AB accelerabitur, quoniam id tendit ad augendum augulum AKP $= \zeta$: momentum autem vis $Bk = B$ erit $= Bb\sin\zeta$, eoque motus rotatorius articuli AB retardabitur, quoniam id tendit ad angulum AKP $= \zeta$ minuendum. Unde ex legibus ſollicitationum erit:

$$2\,Kkkdd\zeta = \mathfrak{B}bdt^2\cos\zeta - Bbdt^2\sin\zeta$$

Deinde momentum vis $B\lambda = \mathfrak{B}$ reſpectu centri gravitatis L eſt $= \mathfrak{B}\beta\cos\eta$, tenditque ad motum rotatorium articuli BC accelerandum, momentum autem vis $Bl = B$, quod eſt $= B\beta\sin\eta$, retardabit eundem motum rotatorium, unde erit;

$$2\,Llldd\eta = \mathfrak{B}\beta dt^2\cos\eta - B\beta dt^2\sin\eta.$$

His ſollicitationibus ad calculum revocatis, priores æquationes pro motu progreſſivo utriusque centri gravitatis inventæ dabunt,

$$Kddp + L\,ddq = 0 \quad \& \quad Kddx + Lddy = 0$$

unde integrando elicitur

$$Kp + Lq = \mathfrak{F}t + f$$
$$Kx + Ly = \mathfrak{G}t + g$$

Cum igitur ſit $q = p + b\sin\zeta + \beta\sin\eta$ & $y = x + b\cos\zeta + \beta\cos\eta$ erit:

$$(K+L)\,p = \mathfrak{F}t + f - Lb\sin\zeta - L\beta\sin\eta$$
$$(K+L)\,x = \mathfrak{G}t + g - Lb\cos\zeta - L\beta\cos\eta$$

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ſeu $p = \frac{\mathfrak{F}t + f - Lb\sin\zeta - L\beta\sin\eta}{K + L}$

$$x = \frac{\mathfrak{G}t + g - Lb\cos\zeta - L\beta\cos\eta}{K + L}$$

ideoque $ddp = -\frac{Lbdd.\sin\zeta - L\beta dd.\sin\eta}{K + L}$

$$ddq = \frac{Kbdd.\sin\zeta + K\beta dd.\sin\eta}{K + L}$$

$$ddx = -\frac{Lbdd.\cos\zeta - L\beta dd.\cos\eta}{K + L}$$

$$ddy = \frac{Kbdd.\cos\zeta + K\beta dd.\cos\eta}{K + L}$$

Ergo hinc vires $\mathfrak{B}$ & B ita definientur ut ſit:

$$\mathfrak{B}dt^2 = -\frac{2KL}{K+L}\,(bdd.\sin\zeta + \beta dd.\sin\eta)$$

$$Bdt^2 = -\frac{2KL}{K+L}\,(bdd.\cos\zeta + \beta dd.\cos\eta)$$

Qui valores ſi in æquationibus ex ſollicitationibus motus rotatorii utriusque articuli ortis ſubſtituantur, prodibit:

$$Kkkdd\zeta = -\frac{KL}{K+L}\left(\begin{array}{l} +bb\cos\zeta.dd\sin\zeta + b\beta\cos\zeta.dd.\sin\eta \\ -bb\sin\zeta.dd\cos\zeta - b\beta\sin\zeta.dd\cos\eta \end{array}\right)$$

$$Llldd\eta = -\frac{KL}{K+L}\left(\begin{array}{l} +l\beta\cos\eta dd.\sin\zeta + \beta\beta\cos\eta.dd.\sin\eta \\ -l\beta\sin\eta dd.\cos\zeta - \beta\beta\sin\eta.dd.\cos\eta \end{array}\right)$$

Cum ergo ſit $\cos m\, dd.\sin n - \sin m\, dd.\cos n = ddn\cos(m-n) + dn^2\sin(m-n)$ habebuntur iſtæ æquationes.

—(K

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$$-\frac{(K+L)kkdd\zeta}{Lb} = bdd\zeta + \beta dd\eta \operatorname{cof}(\zeta-\eta) + \beta d\eta^2 \operatorname{fin}(\zeta-\eta)$$

$$-\frac{(K+L)lldd\eta}{K\beta} = \beta dd\eta + bdd\zeta \operatorname{cof}(\zeta-\eta) - bd\zeta^2 \operatorname{fin}(\zeta-\eta)$$

ex quibus per integrationem elicitur:

$$-\frac{(K+L)kkd\zeta}{Lb} = bd\zeta + \beta d\eta \operatorname{cof}(\zeta-\eta) + \beta \int d\zeta d\eta \operatorname{fin}(\zeta-\eta)$$

$$-\frac{(K+L)lld\eta}{K\beta} = \beta d\eta + bd\zeta \operatorname{cof}(\zeta-\eta) - b\int d\zeta d\eta \operatorname{fin}(\zeta-\eta)$$

Multiplicetur prior per b poſterior per β & addantur, atque prodibit:

$$-\frac{(K+L)kkd\zeta}{L} - \frac{(K+L)lld\eta}{K} = bbd\zeta + \beta\beta d\eta + b\beta (d\zeta + d\eta) \operatorname{cof}(\zeta-\eta) \pm \int dt \sqrt{f}$$

ſeu $\pm \int dt \sqrt{f} = \frac{(K+L)(Kkkd\zeta + Llld\eta)}{KL} + bbd\zeta + \beta\beta d\eta + b\beta(d\zeta + d\eta)\operatorname{cof}(\zeta-\eta)$

Sit $\zeta + \eta = v$ & $\zeta - \eta = u$, ut ſit $\zeta = \frac{v+u}{2}$ & $\eta = \frac{v-u}{2}$

erit:

$$\pm \int dt \sqrt{f} = \frac{(K+L)kkdv}{2L} + \frac{(K+L)kkdu}{2L} + b\beta du \operatorname{cof} u + \frac{(K+L)lldv}{2K} - \frac{(K+L)lldu}{2K}$$

$$+\frac{bbdv}{2}+\frac{bbdu}{2}$$

$$+\frac{\beta\beta dv}{2}-\frac{\beta\beta du}{2}$$

Ponatur porro $\frac{fdt\sqrt{f}}{b\beta}=\frac{dt}{\sqrt{f}}$ atque

$$\frac{(K+L)kk}{2Lb\beta}+\frac{(K+L)ll}{2Kb\beta}+\frac{b}{2\beta}+\frac{\beta}{2b}=m$$

$$\frac{(K+L)kk}{2Lbc}-\frac{(K+L)ll}{2Kbc}+\frac{b}{2c}-\frac{c}{2b}=n \text{ erit.}$$

$$\frac{dt}{\sqrt{f}}=m\,dv+n\,du+dv\cos u$$

ſuperiores vero æquationes differentio-differentiales in has formas transmutentur.

$$0=\frac{(K+L)kk\,dd\zeta}{Lbc}+\frac{b}{c}\,dd\zeta+dd\eta\cos(\zeta-\eta)+d\eta^2\sin(\zeta-\eta)$$

$$0=\frac{(K+L)ll\,dd\eta}{Kbc}+\frac{c}{b}\,dd\eta+dd\zeta\cos(\zeta-\eta)-d\zeta^2\sin(\zeta-\eta)$$

quæ invicem ſubtractæ dabunt:

$$0=nddv+mddu-ddu\cos u+\tfrac{1}{2}(dv^2+du^2)\sin u$$

Prior vero $\frac{dt}{\sqrt{f}}=mdv+ndu+dv\cos u$ ſuppeditat

$$dv=\frac{dt:\sqrt{f}-ndu}{m+\cos u}\ \&\ ddv=\frac{-nddu}{m+\cos u}+\frac{dt\,du\sin u:\sqrt{f}-ndu^2\sin u}{(m+\cos u)^2}$$

atque

atque $\frac{1}{2}dv^2 \operatorname{ſin} u = \frac{\frac{1}{2}dt^2 \operatorname{ſin} u : f - ndtdu \operatorname{ſin} u : \sqrt{f} + \frac{1}{2}n^2 du^2 \operatorname{ſin} u}{(m + \operatorname{coſ} u)^2}$

quibus valoribus ſubſtitutis obtinebitur:

$$o = \frac{-nnddu}{m + \operatorname{coſ} u} + \frac{ndtdu \operatorname{ſin} u : \sqrt{f} - nndu^2 \operatorname{ſin} u}{(m + \operatorname{coſ} u)^2}$$

$$+ mddu \quad + \frac{\frac{1}{2}dt^2 \operatorname{ſin} u : f - ndtdu \operatorname{ſin} u : \sqrt{f} + \frac{1}{2}n^2 du^2 \operatorname{ſin} u}{(m + \operatorname{coſ} u)^2}$$

$$- ddu \operatorname{coſ} u \qquad + \frac{1}{2}du^2 \operatorname{ſin} u$$

quæ reducitur ad hanc:

$$o = (mm - nn)ddu - ddu \operatorname{coſ} u^2 + \frac{\frac{1}{2}dt^2 \operatorname{ſin} u : f - \frac{1}{2}n^2 du^2 \operatorname{ſin} u}{m + \operatorname{coſ} u}$$

$$+ \frac{1}{2} m du^2 \operatorname{ſin} u + \frac{1}{2} du^2 \operatorname{ſin} u \operatorname{coſ} u$$

ſeu $o = 2(mm - nu)ddu - 2ddu \operatorname{coſ} u^2 + du^2 \operatorname{ſin} u \operatorname{coſ} u$

$$+ \frac{dt^2 \operatorname{ſin} u : f + (m^2 - n^2)du^2 \operatorname{ſin} u + mdu^2 \operatorname{ſin} u \operatorname{coſ} u}{m + \operatorname{coſ} u}$$

Sit $dt = \frac{du}{\sqrt{\omega}}$ erit $ddt = o = \frac{ddu}{\sqrt{\omega}} - \frac{dud\omega}{2\omega\sqrt{\omega}}$, ideoque

$ddu = \frac{dud\omega}{2\omega}$, quibus valoribus pro ddu & dt ſubſtitutis fiet

$$o = (mm - nn)\frac{d\omega}{\omega} - \frac{d\omega \operatorname{coſ} u}{\omega} + du \operatorname{ſin} u \operatorname{coſ} u +$$

$$\frac{du \operatorname{ſin} u : f\omega + (m^2 - n^2)du \operatorname{ſin} u + mdu \operatorname{ſin} u \operatorname{coſ} u}{m + \operatorname{coſ} u}$$

ſive

ſive

$$\frac{(mm - nn)\,d\omega - d\omega\,\mathrm{cof}\,u^2 + \omega du\,\mathrm{ſin}\,u\,\mathrm{coſ}\,u}{m + \mathrm{coſ}\,u} +$$

$$\frac{du\,\mathrm{ſin}\,u : f + (m^2 - n^2)\,\omega du\,\mathrm{ſin}\,u + m\omega du\,\mathrm{ſin}\,u\,\mathrm{coſ}\,u}{(m + \mathrm{coſ}\,u)^2} = 0$$

cujus integrale eſt

$$\frac{(mm - nn)\,\omega}{m + \mathrm{coſ}\,u} - \frac{\omega\,\mathrm{coſ}\,u^2}{m + \mathrm{coſ}\,u} + \frac{1}{f(m + \mathrm{coſ}\,u)} = \frac{1}{g}.$$

hinc fit $\omega = \dfrac{f(m + \mathrm{coſ}\,u) - g}{fg(mm - nn - \mathrm{coſ}\,u^2)}$ atque

$dt = \dfrac{du\sqrt{fg(mm - nn - \mathrm{coſ}\,u^2)}}{\sqrt{(mf - g + f\,\mathrm{coſ}\,u)}}$ quo invento habebitur

$dv = \dfrac{dt : \sqrt{f} - ndu}{m + \mathrm{coſ}\,u}$ ſicque per unicam variabilem u determinabuntur t, v, porroque anguli ζ & η, quibus inventis reliquæ quantitates p, q, x, & y innoteſcent, ex quibus non ſolum poſitio corporis, ſed etiam ejus motus definitur. Q. E. J.

Coroll. 1.

41. Ex æquationibus $K\,ddp + L\,ddq = 0$ & $K\,ddx + L\,ddy = 0$ intelligitur corporis ABC centrum gravitatis uniformiter in directum progredi.

Coroll. 2.

42. Eandem vero quoque vivarum quantitatem conſervari, hoc modo patebit:

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$$\frac{2K\,dp\,ddp}{dt^2} = \mathfrak{B}\,dp;\quad \frac{2K\,dx\,ddx}{dt^2} = B\,dx$$

$$\frac{2L\,dq\,ddq}{dt^2} = -\mathfrak{B}\,dq;\quad \frac{2K\,dy\,ddy}{dt^2} = -B\,dy$$

Eſt vero $dp - dq = -bd.\,\text{ſin}\,\zeta - cd.\,\text{ſin}\,\eta$

$dx - dy = -bd.\,\text{coſ}\,\zeta - cd\,\text{coſ}\,\eta$

Ergo $\frac{K(dp^2+dx^2)+L(dq^2+dy^2)}{dt^2} = -\int\mathfrak{B}(bd.\,\text{ſin}\,\zeta + bd.\,\text{ſin}\,\eta) - \int B(bd.\,\text{coſ}\,\zeta + cd.\,\text{coſ}\,\eta)$

Porro vero eſt $\frac{K\,kkd\zeta^2}{dt^2} = \int\mathfrak{B}\,bd.\,\text{ſin}\,\zeta + \int B\,bd.\,\text{coſ}\,\zeta$ atque $\frac{L\,lld\eta^2}{dt^2} = \int\mathfrak{B}\,cd.\,\text{ſin}\,\eta + \int B\,cd.\,\text{coſ}\,\eta$; quibus in unam ſummam collectis erit.

$$\frac{K(dp^2 + dx^2 + kkd\zeta^2) + L(dq^2 + dy^2 + lld\eta^2)}{dt^2} = \text{Conſtanti}$$

At vero iſta expreſſio exhibet vim vivam totius corporis, nam $\frac{K(dp^2 + dx^2 + kkd\zeta^2)}{dt^2}$ eſt vis viva articuli A B; atque $\frac{L(dq^2 + dy^2 + lld\eta^2)}{dt^2}$ eſt vis viva articuli BC.

Scholion.

43. Cum generaliter æquatio ultimo inventa $dt = \frac{du\sqrt{fg}(mm - nn - \text{coſ}\,u^2)}{\sqrt{(mf - g + f\,\text{coſ}\,u)}}$ integrationem non admittat,

caſus ſunt perpendendi, quibus integratio ſuccedit, atque ideo motus commodius definiri queat. Sic enim obtinebimus, ut quoties exemplum proponitur ad unum horum caſuum pertinens, ſolutionem facilius exhibere queamus. Occurrunt autem potiſſimum tres caſus ſequentes.

Caſus. I.

44. Æquationi ſcilicet $dt = \frac{du\sqrt{fg(mm-nn-\cos u^2)}}{\sqrt{(mf-g+f\cos u)}}$ primum ſatisfit, ſi $mf-g+f\cos u = 0$, unde erit $\cos u = \frac{g-mf}{f}$. Hic ergo caſus locum habet, ſi angulus $u = \zeta - \eta$ maneat conſtans, ſeu ſi corpus ABC initio ita fuerit projectum, ut angulus ABC non varietur. Tum igitur, quaſi nullam in B haberet juncturam, inſtar corporis rigidi uniformiter in directum feretur. Erit autem $dv = \frac{dt:\sqrt{f}}{m+\cos u} = \frac{dt\sqrt{f}}{g}$, & $v = \frac{t\sqrt{f}}{g}$; unde ſi ponatur $u = 2i$ fiet $\zeta = \alpha t + i$ & $\eta = \alpha t - i$ poſito $\frac{\sqrt{f}}{g} = 2\alpha$. Ex his elicitur:

$$p = \frac{\mathfrak{F}t + \mathfrak{f} - Lb\sin(\alpha t+i) - Lc\sin(\alpha t-i)}{K+L}$$

$$q = \frac{\mathfrak{F}t + \mathfrak{f} + Kb\sin(\alpha t+i) + Kc\sin(\alpha t-i)}{K+L}$$

$$x = \frac{\mathfrak{G}t + \mathfrak{g} - Lb\cos(\alpha t+i) - Lc\cos(\alpha t-i)}{K+L}$$

$y =$

$$y = \frac{\mathfrak{G}t + \mathfrak{g} + \mathrm{K}b\cos(\mathfrak{a}t+i) + \mathrm{K}c\cos(\mathfrak{a}t-i)}{\mathrm{K}+\mathrm{L}}$$

Hinc ergo erunt celeritates punctorum K & L ſecundum directiones axium $O o$ & $O \omega$.

$$\frac{dp}{dt} = \frac{\mathfrak{F} - \mathrm{L}\mathfrak{a}b\cos(\mathfrak{a}t+i) - \mathrm{L}\mathfrak{a}c\cos(\mathfrak{a}t-i)}{\mathrm{K}+\mathrm{L}}$$

$$\frac{dq}{dt} = \frac{\mathfrak{F} + \mathrm{K}\mathfrak{a}b\cos(\mathfrak{a}t+i) + \mathrm{K}\mathfrak{a}c\cos(\mathfrak{a}t-i)}{\mathrm{K}+\mathrm{L}}$$

$$\frac{dx}{dt} = \frac{\mathfrak{G} + \mathrm{L}\mathfrak{a}b\sin(\mathfrak{a}t+i) + \mathrm{L}\mathfrak{a}c\sin(\mathfrak{a}t-i)}{\mathrm{K}+\mathrm{L}}$$

$$\frac{dy}{dt} = \frac{\mathfrak{G} - \mathrm{K}\mathfrak{a}b\sin(\mathfrak{a}t+i) - \mathrm{K}\mathfrak{a}c\sin(\mathfrak{a}t-i)}{\mathrm{K}+\mathrm{L}}$$

At veto celeritas rotatoria utriusque articuli erit

articuli AB $= \mathfrak{a}$

articuli BC $= \mathfrak{a}$.

Si ergo ambobus articulis initio æquales motus rotatorii imprimantur, inſuperque celeritates progreſſivæ punctorum K & L ejusmodi fuerint, ut in expreſſionibus inventis contineantur, tum corpus quaſi nullam haberet flexuram promovebitur. Si centrum gravitatis totius corporis quieſcere aſſumatur, erunt $\mathfrak{F} = 0$ & $\mathfrak{G} = 0$, atque ſi id in puncto O exiſtat, fiet ſimul $\mathfrak{f} = 0$ & $\mathfrak{g} = 0$. Quod ſi ergo ponamus corpus ABC initio ſuper linea $O o$ in directum jacuiſſe, quia tum erat $t = 0$, debebit eſſe $i = 90°$, & celeritates ſecundum directionem $O o$ evaneſcent, reliquæ vero in directione $O \omega$ erunt

$$\frac{dx}{dt} = \frac{\mathrm{L}\mathfrak{a}b - \mathrm{L}\mathfrak{a}c}{\mathrm{K}+\mathrm{L}} \quad \& \quad \frac{dy}{dt} = -\frac{\mathrm{K}\mathfrak{a}b + \mathrm{K}\mathfrak{a}c}{\mathrm{K}+\mathrm{L}}$$

Caſus II.

45. Hic caſus ſecundus locum habet ſi fuerit $f = \infty$ & $n = 0$, hoc eſt ſi fuerit

$$(K+L)(Kkk - Lll) = KL(cc - bb)$$

tum autem fiet $dv = 0$, ideoque $v =$ conſtanti. Sit ergo $v = 2i$, fietque $dt = \frac{du\sqrt{g(mm - \cos u^2)}}{\sqrt{(m+\cos u)}} = du\sqrt{g}$

ig. 10. $(m - \cos u)$. Quoniam v eſt conſtans, recta angulum KBL biſecans, quæ ſit MBN, cum axe Oo angulum conſtantem ONB perpetuo conſtituet: cum enim ſit KBL $= 180^\circ - \zeta + \eta$, erit KBM $= 90^\circ - \frac{1}{2}\zeta + \frac{1}{2}\eta$ & MB$b = 90^\circ + \frac{1}{2}v$, ergo OMB $= \frac{1}{2}v = i$.

Quoties ergo hujusmodi corpus, in quo eſt

$$K(Kkk + Lkk + Lbb) = L(Kll + Lll + Kcc)$$

initio ita projiciatur, ut poſitio rectæ angulum KBL biſecantis non varietur, tum motus ad hunc caſum ſecundum pertinebit, atque linea MN perpetuo eandem inclinationem ad axem conſervabit. Cum deinde ſit angulus KBL $= 180 - \zeta + \eta = 180 - u$, ex quovis angulo KBL, quem inter motum induit, definiri poterit tempus ab initio elapſum $t = \int du\sqrt{g}(m - \cos u)$. Quia ergo eſt $\zeta = \frac{v+u}{2}$ & $\eta = \frac{v-u}{2}$ erit $\frac{d\zeta}{dt} = \frac{du}{2dt} = \frac{1}{2\sqrt{g}(m - \cos u)}$ & $\frac{d\eta}{dt} = \frac{-1}{2\sqrt{g}(m - \cos u)}$, quarum illa exprimit celeritatem rotatoriam articuli AB, hæc vero articuli BC. Præ-

terea

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terea vero ſi ponamus centrum gravitatis in puncto O quieſcens, erit

$$p = -\frac{L(b\sin\zeta + c\sin\eta)}{K+L};\; x = -\frac{L(b\cos\zeta + c\cos\eta)}{K+L}$$

$$q = \frac{K(b\sin\zeta - c\sin\eta)}{K+L};\; y = \frac{K(b\cos\zeta - c\cos\eta)}{K+L}$$

Celeritates vero punctorum K & L erunt

$$\frac{dp}{dt} = -\frac{L(b\cos\zeta - c\cos\eta)}{2(K+L)\sqrt{g(m\cdot\cos u)}};\; \frac{dx}{dt} = \frac{L(b\sin\zeta - c\sin\eta)}{2(K+L)\sqrt{g(m\cdot\cos u)}}$$

$$\frac{dq}{dt} = \frac{K(b\cos\zeta - c\cos\eta)}{2(K+L)\sqrt{g(m\cdot\cos u)}};\; \frac{dy}{dt} = -\frac{K(b\sin\zeta - c\sin\eta)}{2(K+L)\sqrt{g(m-\cos u)}}$$

Si initio corpus ABC ſuper axe Oo in directum jacuerit, ita ita ut fuerit $\zeta = 90^\circ$ & $\eta = 90^\circ$, ideoque $v = 180^\circ$, & $u = 0$ celeritates ſecundum axem Oo evaneſcent; at ſecundum axem $O\omega$ erit

celeritas puncti K $= \dfrac{L(b-c)}{2(K+L)\sqrt{gm}}$

celeritas puncti L $= -\dfrac{K(b-c)}{2(K+L)\sqrt{gm}}$

& celeritas rotatoria articuli AB $= \dfrac{1}{2\sqrt{gm}}$

celeritas rotatoria articuli BC $= \dfrac{-1}{2\sqrt{gm}}$

Caſus. III.

46. Sit utriusque articuli univerſa materia in ejus

centro

centro gravitatis concentrata, erit $k = 0$ & $l = 0$; unde fit $m = \frac{b}{2c} + \frac{c}{2b}$; $n = \frac{b}{2c} - \frac{c}{2b}$ & $m^2 - n^2 = 1$.

Habebimus ergo $dt = \frac{du \, \text{ſin}\, u. \sqrt{fg}}{\sqrt{(mf - g + f \, \text{coſ}\, u)}}$; cujus integrale eſt

$$t = \sqrt{h} - \frac{2\sqrt{g}}{\sqrt{f}} \sqrt{(mf - g + f \, \text{coſ}\, u)}$$

unde fit $2\sqrt{g}(mf - g + f \, \text{coſ}\, u) = \sqrt{fh} - t\sqrt{f}$ &
$4g(mf - g + f \, \text{coſ}\, u) = fh - 2ft\sqrt{h} + ftt$ ideoque

$$\text{coſ}\, u = \frac{4gg - mfg + fh - ft\sqrt{h} + ftt}{4fg}.$$ Deinde vero erit

$$dv = \frac{4g dt \sqrt{f}}{4gg + (\sqrt{fh} - t\sqrt{f})^2} - \frac{n du}{m + \text{coſ}\, u}.$$

Partis prioris integrale eſt $= -2 \, \text{Atang.} \frac{\sqrt{fg} - t\sqrt{f}}{2g}$, ad poſterius integrale inveniendum ponatur $\text{coſ}\, u = s$, erit $du = \frac{-ds}{\sqrt{(1 - ss)}}$ & $\frac{-ndu}{m + \text{coſ}\, u} = \frac{nds}{(m+s)\sqrt{(1 - ss)}}$; fit $s = \frac{2r}{1 + rr}$, erit $\sqrt{(1 - ss)} = \frac{1 - rr}{1 + rr}$ & $ds = \frac{2dr(1 - rr)}{(1 + rr)^2}$ atque $\frac{ds}{\sqrt{(1 - ss)}} = \frac{2dr}{1 + rr}$; unde fit $\frac{nds}{(m+s)\sqrt{(1 - ss)}} = \frac{2ndr}{m + 2r + mrr}$; cujus integrale eſt $2 \, \text{A tag} \frac{nr}{m + r}$ ob $n = \sqrt{(mm - 1)}$. Ergo ob $rr = \frac{1 - \sqrt{(1 - ss)}}{1 + \sqrt{(1 - ss)}} = \frac{(1 - \sqrt{(1 - ss)})^2}{ss}$; erit

erit $r = \frac{1 - \sqrt{(1 - ss)}}{s} = \frac{1 - \text{ſin}\, u}{\text{coſ}\, u}$, atque $\int \frac{-ndu}{m + \text{coſ}\, u}$

$= 2\,\text{A tag}\, \frac{n - n\,\text{ſin}\, u}{m\,\text{coſ}\, u + 1 - \text{ſin}\, u} = 2\,\text{A tag.}\, \frac{n}{1 + m\,\text{tag}(45° + \frac{1}{2}u)}$.

Quapropter erit

$$v = -2\,\text{A tág}\sqrt{\frac{mf - g + f\,\text{coſ}\, u}{g}} + \text{A tág.}\, \frac{n(1 - \text{ſin}\, u)}{m\,\text{coſ}\, u + 1 - \text{ſin}\, u} \text{ vel}$$

$$v = -2\,\text{A tág}\sqrt{\frac{mf - g + f\,\text{coſ}\, u}{g}} + \text{A tág}\, \frac{n(1 - \text{ſin}\, u + m\,\text{coſ}\, u)}{1 + mu\,\text{ſin}\, u + m\,\text{coſ}\, u}$$

Ceterum cum ſit $\text{coſ}\, u = \frac{4gg - 4mfg + f(\sqrt{h} - t)^2}{4fg}$:patet t non ultra certum limitem augeri poſſe: Cumprimum enim iſta fractio unitatem ſuperat, angulus u fit imaginarius ſicque motus continuatio ceſſare debebit, qui caſus maxime eſt notabilis.

Ponamus initio quo $t = 0$, ambos articulos AB & BC in directum jacuiſſe, ſeu fuiſſe $\eta = \zeta$, ideoque $u = 0$, & $\text{coſ}\, u = 1$: erit ergo $\sqrt{h} = \frac{2\sqrt{g}}{\sqrt{f}} \sqrt{(mf - g + f)}$ ideoque

$$t = \frac{2\sqrt{g}}{\sqrt{f}} \sqrt{(mf - g + f)} - \sqrt{(mf - g + f\,\text{coſ}\, u)})$$

creſcente ergo tempore creſcet angulus u, neque vero ultra duos rectos augeri poteſt, quia tum t iterum decreſceret: motus ergo eousque tantum continuabitur, quoad fiat $u = 180$ & $\text{coſ}\, u = -1$, tantumque tempus, quo motus durabit, erit

$$t = \frac{2\sqrt{g}}{\sqrt{f}} (\sqrt{(mf - g + f)} - \sqrt{(mf - g - f)})$$

Quod

Quod paradoxon reſolvetur, ſi ad celeritatem rotatoriam attendatur: cum enim ſit celeritas rotatoria articuli AB $= \frac{d\zeta}{dt} = \frac{dv + du}{2dt}$ ob $\frac{du}{dt} = \frac{\sqrt{(mf - g + f\operatorname{coſ} u}}{\operatorname{ſin} u . \sqrt{fg}}$ fiet ea, ſi ſin $u = 0$, infinita, ex quo intelligitur hoc caſu in expreſſione $mm - nn$ terminos continentes kk & ll, etiamſi minimi ſtatuantur, negligi non poſſe. Quocirca motus tantum definiri poterit, quamdiu ſin u non eſt $= 0$. Si igitur initio motus ſtatuatur $u = 0$, tum neque ipſum motus initium recte definitur, neque per hunc calculum motum eousque proſequi licet, quoad fiat $u = 180°$. Quamdiu autem u non ſit $= 180$, motus recte aſſignatur eritque

$$\sqrt{(mf - g + f\operatorname{coſ} u)} = \sqrt{(mf - g + f)} - \frac{t\sqrt{f}}{2\sqrt{g}} \text{ ſeu}$$

$$f\operatorname{coſ} u = f - \frac{t\sqrt{f(mf - g + f)}}{\sqrt{g}} + \frac{ftt}{4g} \text{ ideoque}$$

$$\operatorname{coſ} u = 1 - \frac{t\sqrt{(mf - g + f)}}{\sqrt{fg}} + \frac{tt}{4g}.$$

Cum deinde ſit $\frac{du}{dt} = \frac{\sqrt{(mf - g + f\operatorname{coſ} u)}}{\operatorname{ſin} u . \sqrt{fg}}$ & $\frac{dv}{dt} =$

$$\frac{1}{(m + \operatorname{coſ} u)\sqrt{f}} - \frac{ndu}{(m + \operatorname{coſ} u)dt} = \frac{1}{(m + \operatorname{coſ} u)\sqrt{f}} -$$

$\frac{n\sqrt{(mf - g + f\operatorname{coſ} u)}}{\operatorname{ſin} u(m + \operatorname{coſ} u)\sqrt{fg}}$, hinc ad quodvis tempus motus rotatorius utriusque articuli aſſignabitur. Hincque porro per ſolutionem generalem celeritates veras utriusque centri gravitatis K & L definire licet.

Scho-

Scholion.

47. Generaliter autem ex ſtatu ac motu initiali conſtantes f & g, quæ in ſolutione inſunt, definiri ſicque ſolutio ad quemvis caſum particularem accommodari poteſt. Ponamus, quo omnes caſus reduci poſſunt, commune centrum gravitatis in puncto O quieſcere, ita ut fiat $\mathfrak{F}=0$, $\mathfrak{f}=0$ $\mathfrak{G}=0$, & $\mathfrak{g}=0$. Deinde aſſumamus initio ambos articulos indirectum fuiſſe ſitos, ita ut fuerit $\zeta=\eta$ & $\kappa=0$, & quoniam poſitio axis Oo eſt arbitraria, ſumamus initio lineam rectam ABC in axem $O\omega$ incidiſſe, ita ut fuerit $\zeta=0$, & $\eta=0$, ideoque & $v=0$. Fuit ergo initio Fig. 11.

$$OK=\frac{Lb+L\beta}{K+L} \;\&\; OL=\frac{Kb+K\beta}{K+L}.$$

Deinde cum ſit $\frac{du}{dt}=\frac{\sqrt{(mf-g+f\cos u)}}{\sqrt{fg(mm-nn-\cos u)^2}}$ & $\frac{dv}{dt}=$

$$\frac{1}{(m+\cos u)\sqrt{f}}-\frac{n\,du}{(m+\cos u)dt}=\frac{1}{(m+\cos u)\sqrt{f}}-\frac{n\sqrt{(mf-g+f\cos u)}}{(m+\cos u)\sqrt{fg(mm-nn-\cos u^2)}}$$

ob $\zeta=\frac{v+u}{2}$ & $\eta=\frac{v-u}{2}$ erit

$$\frac{d\zeta}{dt}=\frac{1}{2(m+\cos u)\sqrt{f}}+\frac{(m-n+\cos u)\sqrt{(mf-g+f\cos u)}}{2(m+\cos u)\sqrt{fg(mm-nn-\cos u^2)}}$$

$$\frac{d\eta}{dt}=\frac{1}{2(m+\cos u)\sqrt{f}}-\frac{(m+n+\cos u)\sqrt{(mf-g+f\cos u)}}{2(m+\cos u)\sqrt{fg(mm-nn-\cos u^2)}}$$

Initio igitur quo erat $u=0$, fuit

$$\frac{d\zeta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m-n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg}(m^2-n^2-1)}$$

$$\frac{d\eta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m+n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg}(m^2-n^2-1)}.$$

Cum igitur punctorum **K** & **L** celeritates ſint:

	in directione Oo	in directione $O\omega$
K;	$\frac{dp}{dt} = -\frac{Lbd\zeta\cos\zeta - L\beta d\eta\cos\eta}{(K+L)dt}$	$\frac{dx}{dt} = \frac{Lbd\zeta\sin\zeta + L\beta d\eta\sin\eta}{(K+L)dt}$
L;	$\frac{dq}{dt} = \frac{Kbd\zeta\cos\zeta + K\beta d\eta\cos\eta}{(K+L)dt}$	$\frac{dy}{dt} = -\frac{Kbd\zeta\sin\zeta - K\beta d\eta\sin\eta}{(K+L)dt}$

initio celeritates punctorum ſecundum directionem $O\omega$ evanuerunt. In directione autem Oo erant celeritates inter ſe ut L ad K & altera alterius erat negativa. Quod ſi ergo ponamus celeritatem puncti K in directione Kk fuiſſe $= \frac{L\sqrt{h}}{K+L}$ & celeritatem puncti L in directione Ll fuiſſe $= \frac{K\sqrt{h}}{K+L}$ erit $\sqrt{h} = \frac{bd\zeta}{dt} + \frac{\beta d\eta}{dt}$ ſeu

$$\sqrt{h} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{(b-\beta)\sqrt{(mf-g+f)}}{2\sqrt{fg}(m^2-n^2-1)} - \frac{n(b+\beta)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg}(m^2-n^2-1)}$$

Pona-

Ponamus:

$$\frac{1}{2(m+1)\sqrt{f}} - \frac{n\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(mm-nn-1)}} = \frac{\mu}{\sqrt{f}}$$

$$\&\quad \frac{\sqrt{(mf-g+f)}}{2\sqrt{fg(mm-nn-1)}} = \frac{\nu}{\sqrt{f}}.$$

eratque initio celeritas rotatoria circa centrum gravitatis

Articuli AB $= \dfrac{\mu+\nu}{\sqrt{f}}$

articuli BC $= \dfrac{\mu-\nu}{\sqrt{f}}$

& celeritas puncti K ſecundum $Kk = \dfrac{L((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

& celeritas puncti L ſecundum $Ll = \dfrac{K((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

Erit autem $\dfrac{\mu}{\sqrt{f}} + \dfrac{n\nu}{(m+1)\sqrt{f}} = \dfrac{1}{2(m+1)\sqrt{f}}$, ideoque $2(m+1)\mu + 2n\nu = 1$; ergo $\nu = \dfrac{1}{2n} - \dfrac{(m+1)\mu}{n}$ ſeu $\mu = \dfrac{1-2n\nu}{2(m+1)}$.

Quia porro eſt $\sqrt{(mf-g+f)} = 2\nu\sqrt{g(mm-nn-1)}$ erit $(m+1)f-g = 4\nu^2 g(mm-nn-1)$; ideoque $g = \dfrac{(m+1)f}{1+4\nu^2(m^2-n^2-1)}$.

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Cum deinde ſit $\sqrt{h} = \frac{(b+\beta)\mu + (b-\beta)\nu}{\sqrt{f}}$; fiet

$$\sqrt{f} = \frac{(b+\beta)\mu + (b-\beta)\nu}{\sqrt{h}} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{\nu b(m-n+1) - \nu\beta(m+n+1)}{(m+1)\sqrt{h}}$$

ſeu $\sqrt{f} = \frac{b+\beta}{2(m+1)\sqrt{h}} = \frac{\nu(K+L)(Lbll - K\beta kk)}{KLb\beta(m+1)\sqrt{h}}$

eſtque $m^2 - n^2 - 1 = \frac{K+L}{KLb^2\beta^2}((K+L)k^2 ll + K\beta^2 k^2 + Lbbll)$

Hinc ergo ex motu initiali definiuntur litteræ f & g, quibus cognitis deinceps ad quodvis tempus motûs & ſitus corporis determinabitur.

Problema. VII.

Fig. 12. 48. *Conſtet corpus ex quotcunque articulis AB, BC, CD, DE &c. flexuris in B, C, D, &c. invicem conjunctis, quæriturque motus, quo hoc corpus ſuper plano politiſſimo horizontali ſit progreſſurum, poſtquam ipſi ſemel motus quicunque fuerit impreſſus.*

Solutio.

Sumtis pro lubitu in plano horizontali duobus axibus orthogonalibus Oo, $O\omega$, ad quos quovis momento poſitio corporis referatur, perveneritque elapſo tempore t in ſitum, quem figura repræſentat. Sint ſingulorum articulorum centra gravitatis in K, L, M, N &c. unde ad

axem

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axem Oo demittantur perpendicula KP, LQ, MR, NS, &c. & vocentur:

$AK = a$; $BK = b$; $BL = \beta$; $CL = c$; $CM = \gamma$; $DM = d$;
$DN = \delta$ &c.

$OP = p$; $OQ = q$; $OR = r$; $OS = s$ &c.

$PK = x$; $QL = y$; $RM = z$; $SN = v$ &c.

Anguli: $AKP = \zeta$; $BLQ = \eta$; $CMR = \theta$; $DNS = \iota$; &c.

ex quibus oriuntur ſequentes æquationes:

$q - p = b\,\text{ſin}\,\zeta + \beta\,\text{ſin}\,\eta$	$y - x = b\,\text{coſ}\,\zeta + \beta\,\text{coſ}\,\eta$
$r - q = c\,\text{ſin}\,\eta + \gamma\,\text{ſin}\,\theta$	$z - y = c\,\text{coſ}\,\eta + \gamma\,\text{coſ}\,\theta$
$s - r = d\,\text{ſin}\,\theta + \delta\,\text{ſin}\,\iota$	$v - z = d\,\text{coſ}\,\theta + \delta\,\text{coſ}\,\iota$
&c.	&c.

Deinde ſi punctorum K, L, M, N &c. motus reſolvantur in laterales ſecundum directiones axium Oo, $O\omega$, erit

Celeritas puncti	in directione Oo	in directione $O\omega$
K	$= \frac{dp}{dt}$	$= \frac{dx}{dt}$
L	$= \frac{dq}{dt}$	$= \frac{dy}{dt}$
M	$= \frac{dr}{dt}$	$= \frac{dz}{dt}$
N	$= \frac{ds}{dt}$	$= \frac{dv}{dt}$
	&c.	&c.

Celeritates vero angulares cujusque articuli circa ſuum gravitatis centrum ex incrementis angulorum ζ, η, θ, &c. defi-

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definientur, ita ut in diſtantia α quovis centro gravitatis $= 1$ ſit;

Celeritas rotatoria

Articuli AB circa K $= \frac{d\zeta}{dt}$

Articuli BC circa L $= \frac{d\eta}{dt}$

Articuli CD circa M $= \frac{d\theta}{dt}$

Articuli DE circa N $= \frac{d\iota}{dt}$

&c.

Si denique:	Maſſa	Momentum inertiæ
Articuli AB	$= K$	$= Kkk$
Articuli BC	$= L$	$= Lll$
Articuli CD	$= M$	$= Mmm$
Articuli DE	$= N$	$= Nnn$
&c.	&c.	&c.

Vires, quas juncturæ ſuſtinent, & quibus motus articulorum alterantur, pariter ſecundum directiones Oo & $O\omega$ reſolvantur vocenturque

Vis $Bk = Bl = B$	Vis $B\varkappa = B\lambda = \mathfrak{B}$
Vis $Cl = Cm = C$	Vis $C\lambda = C\mu = \mathfrak{C}$
Vis $Dm = Dn = D$	Vis $D\mu = D\nu = \mathfrak{D}$
&c.	&c.

Iſtis

His igitur motus punctorum K, L, M, N, &c. ita afficientur, ut ſit:

$$2K\,ddp = \mathfrak{B}\,dt^2$$
$$2L\,ddq = \mathfrak{C}\,dt^2 - \mathfrak{B}\,dt^2$$
$$2M\,ddr = \mathfrak{D}\,dt^2 - \mathfrak{C}\,dt^2$$
$$2N\,dds = \cdots - \mathfrak{D}\,dt^2$$
&c.

$$2K\,ddx = B\,dt^2$$
$$2L\,ddy = C\,dt^2 - B\,dt^2$$
$$2M\,ddz = D\,dt^2 - C\,dt^2$$
$$2N\,ddv = \cdots - D\,dt^2$$
&c.

Tum vero ſingulorum articulorum motus rotatorii a momentis harum virium ita accelerabuntur; ut fiat:

$$2Kkk\,dd\zeta = (\mathfrak{B}b\cos\zeta - Bb\sin\zeta)\,dt^2$$
$$2Lll\,dd\eta = (\mathfrak{C}c\cos\eta - Cc\sin\eta)\,dt^2 + (\mathfrak{B}\beta\cos\eta - B\beta\sin\eta)\,dt^2$$
$$2Mmm\,dd\theta = (\mathfrak{D}d\cos\theta - Dd\sin\theta)\,dt^2 + (\mathfrak{C}\gamma\cos\theta - C\gamma\sin\theta)\,dt^2$$
$$2Nnn\,dd\iota = \cdots + (\mathfrak{D}\delta\cos\iota - D\delta\sin\iota)\,dt^2$$
&c.

Ex prioribus ſollicitationibus oriuntur iſtæ binæ æquationes

$$K\,ddp + L\,ddq + M\,ddr + N\,dds + \&c. = 0$$
$$K\,ddx + L\,ddy + M\,ddz + N\,ddv + \&c. = 0$$

quæ bis integratæ dabunt:

$$Kp + Lq + Mr + Ns + \&c. = \mathfrak{F}t + \mathfrak{f}$$
$$Lx + Ly + Mz + Nv + \&c. = \mathfrak{G}t + \mathfrak{g}$$

quibus

quibus communi centri gravitatis totius corporis motus uniformis ſecundum lineam rectam indicatur. Cum ergo ſit

$$q = p + b\,\text{ſin}\,\zeta + \varepsilon\,\text{ſin}\,\eta$$
$$r = p + b\,\text{ſin}\,\zeta + (c + \varepsilon)\,\text{ſin}\,\eta + \gamma\,\text{ſin}\,\theta$$
$$s = p + b\,\text{ſin}\,\zeta + (c + \varepsilon)\,\text{ſin}\,\eta + (d + \gamma)\,\text{ſin}\,\theta + \delta\,\text{ſin}\,\iota$$

&c.

$$y = x + b\,\text{coſ}\,\zeta + \varepsilon\,\text{coſ}\,\eta$$
$$z = x + b\,\text{coſ}\,\zeta + (c + \varepsilon)\,\text{coſ}\,\eta + \gamma\,\text{coſ}\,\theta$$
$$v = x + b\,\text{coſ}\,\zeta + (c + \varepsilon)\,\text{coſ}\,\eta + (d + \gamma)\,\text{coſ}\,\theta + \delta\,\text{coſ}\,\iota$$

&c.

Ponatur maſſa totius corporis $K + L + M + N + \&c. = S$ fietque:

$$p = \mathfrak{F}t + \mathfrak{f}\,\frac{\begin{array}{l} -(S-L)\,b\,\text{ſin}\,\zeta - (S-K-L)\,c\,\text{ſin}\,\eta - (S-K-L-M)\,d\,\text{ſin}\,\theta \\ -(S-L)\,b\,\text{ſin}\,\eta - (S-K-L)\,\gamma\,\text{ſin}\,\theta - (S-K-L-M)\,\delta\,\text{ſin}\,\iota \end{array} \;\&c.}{S}$$

$$q = \mathfrak{F}t + \mathfrak{f}\,\frac{\begin{array}{l} +K\,b\,\text{ſin}\,\zeta - (S-K-L)\,c\,\text{ſin}\,\eta - (S-K-L-M)\,d\,\text{ſin}\,\theta \\ +K\,\varepsilon\,\text{ſin}\,\eta - (S-K-L)\,\gamma\,\text{ſin}\,\theta - (S-K-L-M)\,\delta\,\text{ſin}\,\iota \end{array} \;\&c.}{S}$$

$$r = \mathfrak{F}t + \mathfrak{f}\,\frac{\begin{array}{l} +K\,b\,\text{ſin}\,\eta + (K+L)\,c\,\text{ſin}\,\eta - (S-K-L-M)\,d\,\text{ſin}\,\theta \\ +K\,\varepsilon\,\text{ſin}\,\eta + (K+L)\,\gamma\,\text{ſin}\,\theta - (S-K-L-M)\,\delta\,\text{ſin}\,\iota \end{array} \;\&c.}{S}$$

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$$s = \mathfrak{F}t + \mathfrak{f}\,\frac{\begin{array}{l} +Kb\,\text{ſin}\,\zeta + (K+L)c\,\text{ſin}\,\eta + (K+L+M)d\,\text{ſin}\,\theta \\ +K\varsigma\,\text{ſin}\,\eta + (K+L)\gamma\,\text{ſin}\,\theta + (K+L+M)\delta\,\text{ſin}\,\iota \end{array}\ \&c.}{S}$$

&c.

Similique modo applicatæ $x, y, z, v,$ &c. definiatur prodibitque:

$$x = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{l} -(S-K)b\,\text{coſ}\,\zeta - (S-K-L)c\,\text{coſ}\,\eta \\ \quad -(S-K-L-M)d\,\text{coſ}\,\theta \\ -(S-K)\varsigma\,\text{coſ}\,\eta - (S-K-L)\gamma\,\text{coſ}\,\theta \\ \quad -(S-K-L-M)\delta\,\text{coſ}\,\iota \end{array}\ \&c.}{S}$$

$$y = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{l} +Kb\,\text{coſ}\,\zeta - (S-K-L)c\,\text{coſ}\,\eta \\ \quad -(S-K-L-M)d\,\text{coſ}\,\theta \\ +K\varsigma\,\text{coſ}\,\eta - (S-K-L)\gamma\,\text{coſ}\,\theta \\ \quad -(S-K-L-M)\delta\,\text{coſ}\,\iota \end{array}\ \&c.}{S}$$

$$z = \mathfrak{G}t + \mathfrak{g}\,\frac{\begin{array}{l} +Kb\,\text{coſ}\,\zeta + (K+L)c\,\text{coſ}\,\eta \\ \quad -(S-K-L-M)d\,\text{coſ}\,\theta \\ +K\varsigma\,\text{coſ}\,\eta + (K+L)\gamma\,\text{coſ}\,\theta \\ \quad -(S-K-L-M)\delta\,\text{coſ}\,\iota \end{array}\ \&c.}{S}$$

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$$v = \mathfrak{G}t + g\,\frac{\begin{matrix}+Kb\cos\zeta + (K+L)c\cos\eta + (K+L+M)\\ d\cos\theta\\ +K\epsilon\cos\eta + (K+L)\gamma\cos\theta + (K+L+M)\\ \delta\cos\iota\end{matrix}\ \&c.}{S}$$

&c.

Deinde ex æquationibus, quas ſollicitationes puncto-rum K, L, M, N &c. ſuppeditaverunt, colligitur fore.

$$\frac{2K(dpddp + dxddx) + 2L(dqddq + dyddy) + 2M(drddr + dzddz)\ \&c.}{dt^2}$$

$$= \mathfrak{B}(dp - dq) + \mathfrak{C}(dq - dr) + \mathfrak{D}(dr - ds) + \&c. =$$

$$+ B(dx - dy) + C(dy - dz) + D(dz - dv) + \&c.$$

$$- \mathfrak{B}bd\zeta\cos\zeta - \mathfrak{B}\epsilon d\eta\cos\eta - \mathfrak{C}\gamma d\theta\cos\theta - \mathfrak{D}\delta d\iota\cos\iota - \&c.$$
$$- \mathfrak{C}cd\eta\cos\eta - \mathfrak{D}dd\theta\cos\theta$$

$$+ Bbd\zeta\sin\zeta + B\epsilon d\eta\sin\eta + C\gamma d\theta\sin\theta + D\delta d\iota\sin\iota + \&c.$$
$$+ Ccd\eta\sin\eta + Ddd\theta\sin\theta$$

At ſollicitationes motus rotatorii dabunt

$$\frac{2Kk^2\,d\zeta dd\zeta + 2Ll^2\,d\eta dd\eta + 2Mm^2\,d\theta dd\theta + \&c.}{dt^2} =$$

$\mathfrak{B}$

$$\mathfrak{B}\, b\, d\zeta \cos\zeta + \mathfrak{B}\, c\, d\eta \cos\eta + \mathfrak{C}\, \gamma\, d\theta \cos\theta + \mathfrak{D}\, \delta\, d\iota \cos\iota$$
$$+ \mathfrak{C}\, c\, d\eta \cos\eta + \mathfrak{D}\, d\, d\theta \cos\theta \qquad \&c.$$
$$- B\, b\, d\zeta \sin\zeta - B\, c\, d\eta \sin\eta - C\, \gamma\, d\theta \sin\theta - D\, \delta\, d\iota \sin\iota$$
$$- C\, c\, d\eta \sin\eta - D\, d\, d\theta \sin\theta$$

quæ expreſſio cum præcedenti ſit æqualis & contrariis ſignis affecta ſequitur fore:

$$\frac{K(dp^2 + dx^2 + k^2 d\zeta^2) + L(dq^2 + dy^2 + l^2 d\eta^2) + M(dr^2 + dz^2 + m^2 d\theta^2) + \&c.}{dt^2}$$

$=$ Conſt. ſicque patet pepetuo eandem virium vivarum quantitatem conſervari.

Quærantur denique valores virium $\mathfrak{B}$, B, $\mathfrak{C}$, C, $\mathfrak{D}$, D, &c. quæ erunt:

$$\tfrac{1}{2}\mathfrak{B}\, dt^2 = K\, ddp$$
$$\tfrac{1}{2}\mathfrak{C}\, dt^2 = K\, ddp + L\, ddq$$
$$\tfrac{1}{2}\mathfrak{D}\, dt^2 = K\, ddp + L\, ddq + M\, ddr$$
&c.

$$\tfrac{1}{2}\,B\,dt^2 = K\,ddx$$

$$\tfrac{1}{2}\,C\,dt^2 = K\,ddx + L\,ddy$$

$$\tfrac{1}{2}\,D\,dt^2 = K\,ddx + L\,ddy + M\,ddz$$

&c.

Si igitur valores pro 𝔅, ℭ, 𝔇, &c. & B, C, D, &c. hinc oriundi in æquationibus ex motibus rotatoriis ortis ſubſtituantur; prodibit

$$K\,kk\,dd\zeta = b(K\,ddp\cos\zeta - L\,ddx\sin\zeta)$$

$$L\,ll\,dd\eta = \begin{cases} -K\,c\,ddp\cos\eta + L\,c\,ddq\cos\eta + K\varsigma\,ddp\cos\eta \\ -K\,c\,ddx\sin\eta - L\,c\,ddy\sin\eta - K\varsigma\,ddx\sin\eta \end{cases}$$

$$M\,mm\,dd\theta = \begin{cases} -\cos\theta\,(K(\gamma+d)ddp + L(\gamma+d)ddq + M\,dddr) \\ -\sin\theta\,(K(\gamma+d)\,ddx + L(\gamma+d)\,ddy + M\,dddz) \end{cases}$$

$$N\,nn\,dd\iota = \begin{cases} +\cos\iota\,(K(\delta+e)\,ddp + L(\delta+e)\,ddq + M(\delta+e) \\ \qquad ddr + N\,e\,dds) \\ -\sin\iota\,(K(\delta+e)\,ddx + L(\delta+e)\,ddy + M(\delta+e) \\ \qquad ddz + N\,e\,ddv) \end{cases}$$

&c.

Quod

Quod ſi nunc hic loco p, q, r, &c. & x, y, z, &c. valores ſupra inventi ſubſtituantur, habebuntur tot æquationes differentio-differentiales, quot ſunt anguli ζ, η, θ, &c. per easque ergo iſti anguli determinabuntur ex tempore jam elapſo $= t$, quibus inventis ſimul valores coordinatarum p, x, q, y, r, z, &c. innoteſcent, ſicque ad datum tempus poſitio totius corporis una cum motu cujusque articuli definiri poterit.

Q. E. J.

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5 10 45
8K3 187

Tab. I.

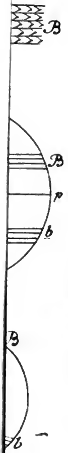


Fig. 9.

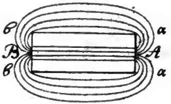
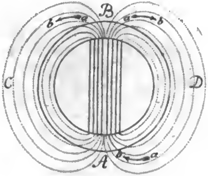


Fig. 8.



F.H. Friſch. ſc.

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$$\frac{d\zeta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m-n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}$$

$$\frac{d\eta}{dt} = \frac{1}{2(m+1)\sqrt{f}} + \frac{(m+n+1)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}.$$

Cum igitur punctorum **K** & **L** celeritates ſint:

	in directione Oo	in directione $O\omega$
K:	$\frac{dp}{dt} = -\frac{Lbd\zeta\cos\zeta - L\beta d\eta\cos\eta}{(K+L)dt}$	$\frac{dx}{dt} = \frac{Lbd\zeta\sin\zeta + L\beta d\eta\sin\eta}{(K+L)dt}$
L;	$\frac{dq}{dt} = \frac{Kbd\zeta\cos\zeta + K\beta d\eta\cos\eta}{(K+L)dt}$	$\frac{dy}{dt} = -\frac{Kbd\zeta\sin\zeta - K\beta d\eta\sin\eta}{(K+L)dt}$

initio celeritates punctorum ſecundum directionem $O\omega$ evanuerunt. In directione autem Oo erant celeritates inter ſe ut L ad K & altera alterius erat negativa. Quod ſi ergo ponamus celeritatem puncti K in directione Kk fuiſſe $= \frac{L\sqrt{h}}{K+L}$

& celeritatem puncti L in directione Ll fuiſſe $= \frac{K\sqrt{h}}{K+L}$

erit $\sqrt{h} = \frac{bd\zeta}{dt} + \frac{\beta d\eta}{dt}$ ſeu

$$\sqrt{h} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{(b-\beta)\sqrt{(mf-g+f)}}{2\sqrt{fg(m^2-n^2-1)}} - \frac{n(b+\beta)\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(m^2-n^2-1)}}$$

Pona-

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Ponamus:

$$\frac{1}{2(m+1)\sqrt{f}} - \frac{n\sqrt{(mf-g+f)}}{2(m+1)\sqrt{fg(mm-nn-1)}} = \frac{\mu}{\sqrt{f}}$$

$$\& \quad \frac{\sqrt{(mf-g+f)}}{2\sqrt{fg(mm-nn-1)}} = \frac{\nu}{\sqrt{f}}.$$

eratque initio celeritas rotatoria circa centrum gravitatis

$$\text{Articuli AB} = \frac{\mu+\nu}{\sqrt{f}}$$

$$\text{articuli BC} = \frac{\mu-\nu}{\sqrt{f}}$$

& celeritas puncti K ſecundum $Kk = \frac{L((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

& celeritas puncti L ſecundum $Ll = \frac{K((b+\beta)\mu+(b-\beta)\nu)}{(K+L)\sqrt{f}}$

Erit autem $\frac{\mu}{\sqrt{f}} + \frac{n\nu}{(m+1)\sqrt{f}} = \frac{1}{2(m+1)\sqrt{f}}$, ideoque $2(m+1)\mu + 2n\nu = 1$; ergo $\nu = \frac{1}{2n} - \frac{(m+1)\mu}{n}$ ſeu $\mu = \frac{1-2n\nu}{2(m+1)}$.

Quia porro eſt $\sqrt{(mf-g+f)} = 2\nu\sqrt{g(mm-nn-1)}$ erit $(m+1)f-g = 4\nu^2 g(mm-nn-1)$; ideoque $g = \frac{(m+1)f}{1+4\nu^2(m^2-n^2-1)}$.

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Cum deinde ſit $\sqrt{h} = \frac{(b+\beta)\mu+(b-\beta)\nu}{\sqrt{f}}$; fiet

$$\sqrt{f} = \frac{(b+\beta)\mu+(b-\beta)\nu}{\sqrt{h}} = \frac{b+\beta}{2(m+1)\sqrt{f}} + \frac{\nu b(m-n+1)-\nu\beta(m+n+1)}{(m+1)\sqrt{h}}$$

ſeu $\sqrt{f} = \frac{b+\beta}{2(m+1)\sqrt{h}} = \frac{\nu(K+L)(Lbll-K\beta kk)}{KLb\beta(m+1)\sqrt{h}}$

eſtque $m^2 - n^2 - 1 = \frac{K+L}{KLb^2\beta^2}((K+L)k^2ll+K\beta^2k^2 + Lbbll)$

Hinc ergo ex motu initiali definiuntur litteræ f & g, quibus cognitis deinceps ad quodvis tempus motûs & ſitus corporis determinabitur.

Problema. VII.

Fig. 12. 48. *Conſtet corpus ex quotcunque articulis AB, BC, CD, DE &c. flexuris in B, C, D, &c. invicem conjunctis, quæriturque motus, quo hoc corpus ſuper plano politiſſimo horizontali ſit progreſſurum, poſtquam ipſi ſemel motus quicunque fuerit impreſſus.*

Solutio.

Sumtis pro lubitu in plano horizontali duobus axibus orthogonalibus $O o$, $O \omega$, ad quos quovis momento poſitio corporis referatur, perveneritque elapſo tempore t in ſitum, quem figura repræſentat. Sint ſingulorum articulorum centra gravitatis in K, L, M, N &c. unde ad

axem

axem $O o$ demittantur perpendicula KP, LQ, MR, NS, &c. & vocentur:

$AK = \alpha$; $BK = b$; $BL = \beta$; $CL = c$; $CM = \gamma$; $DM = d$; $DN = \delta$ &c.

$OP = p$; $OQ = q$; $OR = r$; $OS = s$ &c.

$PK = x$; $QL = y$; $RM = z$; $SN = v$ &c.

Anguli: $AKP = \zeta$; $BLQ = \eta$; $CMR = \theta$; $DNS = \iota$; &c.

ex quibus oriuntur ſequentes æquationes:

$q - p = b \operatorname{ſin} \zeta + \beta \operatorname{ſin} \eta$	$y - x = b \operatorname{coſ} \zeta + \beta \operatorname{coſ} \eta$
$r - q = c \operatorname{ſin} \eta + \gamma \operatorname{ſin} \theta$	$z - y = c \operatorname{coſ} \eta + \gamma \operatorname{coſ} \theta$
$s - r = d \operatorname{ſin} \theta + \delta \operatorname{ſin} \iota$	$v - z = d \operatorname{coſ} \theta + \delta \operatorname{coſ} \iota$
&c.	&c.

Deinde ſi punctorum K, L, M, N &c. motus reſolvantur in laterales ſecundum directiones axium $O o$, $O \omega$, erit

Celeritas puncti	in directione $O o$	in directione $O \omega$
K	$= \frac{dp}{dt}$	$= \frac{dx}{dt}$
L	$= \frac{dq}{dt}$	$= \frac{dy}{dt}$
M	$= \frac{dr}{dt}$	$= \frac{dz}{dt}$
N	$= \frac{ds}{dt}$	$= \frac{dv}{dt}$
	&c.	&c.

Celeritates vero angulares cujusque articuli circa ſuum gravitatis centrum ex incrementis angulorum ζ, η, θ, &c.

 defi-

definientur, ita ut in diſtantia α quovis centro gravitatis $=1$ ſit;

Celeritas rotatoria

Articuli AB circa K $= \frac{d\zeta}{dt}$

Articuli BC circa L $= \frac{d\eta}{dt}$

Articuli CD circa M $= \frac{d\theta}{dt}$

Articuli DE circa N $= \frac{d\iota}{dt}$

&c.

Si denique:	Maſſa	Momentum inertiæ
Articuli AB	$= K$	$= Kkk$
Articuli BC	$= L$	$= Lll$
Articuli CD	$= M$	$= Mmm$
Articuli DE	$= N$	$= Nnn$
&c.	&c.	&c.

Vires, quas junſturæ ſuſtinent, & quibus motus articulorum alterantur, pariter ſecundum directiones Oo & $O\omega$ reſolvantur vocenturque

Vis $Bk = Bl = B$	Vis $B\varkappa = B\lambda = \mathfrak{B}$
Vis $Cl = Cm = C$	Vis $C\lambda = C\mu = \mathfrak{C}$
Vis $Dm = Dn = D$	Vis $D\mu = D\nu = \mathfrak{D}$
&c.	&c.

Iſtis

His igitur motus punctorum K, L, M, N, &c. ita afficientur, ut ſit:

$$2K\,ddp = \mathfrak{B}\,dt^2$$
$$2L\,ddq = \mathfrak{C}\,dt^2 - \mathfrak{B}\,dt^2$$
$$2M\,ddr = \mathfrak{D}\,dt^2 - \mathfrak{C}\,dt^2$$
$$2N\,dds = \cdots - \mathfrak{D}\,dt^2$$
&c.

$$2K\,ddx = B\,dt^2$$
$$2L\,ddy = C\,dt^2 - B\,dt^2$$
$$2M\,ddz = D\,dt^2 - C\,dt^2$$
$$2N\,ddv = \cdots - D\,dt^2$$
&c.

Tum vero ſingulorum articulorum motus rotatorii a momentis harum virium ita accelerabuntur; ut fiat:

$$2K\,kk\,dd\zeta = (\mathfrak{B}\,b\cos\zeta - B\,b\sin\zeta)\,dt^2$$
$$2L\,ll\,dd\eta = (\mathfrak{C}\,c\cos\eta - C\,c\sin\eta)\,dt^2 + (\mathfrak{B}\,\beta\cos\eta - B\,\beta\sin\eta)\,dt^2$$
$$2M\,mm\,dd\theta = (\mathfrak{D}\,d\cos\theta - D\,d\sin\theta)\,dt^2 + (\mathfrak{C}\,\gamma\cos\theta - C\,\gamma\sin\theta)\,dt^2$$
$$2N\,nn\,dd\iota = \cdots + (\mathfrak{D}\,\delta\cos\iota - D\,\delta\sin\iota)\,dt^2$$
&c.

Ex prioribus ſollicitationibus oriuntnr iſtæ binæ æquationes

$$K\,ddp + L\,ddq + M\,ddr + N\,dds + \&c. = 0$$
$$K\,ddx + L\,ddy + M\,ddz + N\,ddv + \&c. = 0$$

quæ bis integratæ dabunt:

$$Kp + Lq + Mr + Ns + \&c. = \mathfrak{F}t + f$$
$$Lx + Ly + Mz + Nv + \&c. = \mathfrak{G}t + \mathfrak{g}$$

quibus

quibus communi centri gravitatis totius corporis motus uniformis ſecundum lineam rectam indicatur. Cum ergo ſit

$$q = p + b \sin\zeta + \epsilon \sin\eta$$
$$r = p + b \sin\zeta + (c + \epsilon) \sin\eta + \gamma \sin\theta$$
$$s = p + b \sin\zeta + (c + \epsilon) \sin\eta + (d + \gamma) \sin\theta + \delta \sin\iota$$

&c.

$$y = x + b \cos\zeta + \epsilon \cos\eta$$
$$z = x + b \cos\zeta + (c + \epsilon) \cos\eta + \gamma \cos\theta$$
$$v = x + b \cos\zeta + (c + \epsilon) \cos\eta + (d + \gamma) \cos\theta + \delta \cos\iota$$

&c.

Ponatur maſſa totius corporis K + L + M + N + &c. = S fietque:

$$p = \mathfrak{F}t + f\frac{\begin{array}{l} -(S-L) b \sin\zeta - (S-K-L) c \sin\eta - (S-K-L-M) d \sin\theta \\ -(S-L) \epsilon \sin\eta - (S-K-L) \gamma \sin\theta - (S-K-L-M) \delta \sin\iota \end{array}}{S} \ \&c.$$

$$q = \mathfrak{F}t + f\frac{\begin{array}{l} +K b \sin\zeta - (S-K-L) c \sin\eta - (S-K-L-M) d \sin\theta \\ +K \epsilon \sin\eta - (S-K-L) \gamma \sin\theta - (S-K-L-M) \delta \sin\iota \end{array}}{S} \ \&c.$$

$$r = \mathfrak{F}t + f\frac{\begin{array}{l} +K b \sin\eta + (K+L) c \sin\eta - (S-K-L-M) d \sin\theta \\ +K \epsilon \sin\eta + (K+L) \gamma \sin\theta - (S-K-L-M) \delta \sin\iota \end{array}}{S} \ \&c.$$

$s =$

$$s = \mathfrak{F}t + \frac{\mathfrak{f}\begin{array}{l} +Kb\sin\zeta + (K+L)c\sin\eta + (K+L+M)d\sin\theta \\ +K\beta\sin\eta + (K+L)\gamma\sin\theta + (K+L+M)\delta\sin\iota \end{array} \&c.}{S}$$

&c.

Similique modo applicatæ x, y, z, v &c. definiatur prodibitque:

$$x = \mathfrak{G}t + \frac{\mathfrak{g}\begin{array}{l} -(S-K)b\cos\zeta - (S-K-L)c\cos\eta \\ \quad -(S-K-L-M)d\cos\theta \\ -(S-K)\beta\cos\eta - (S-K-L)\gamma\cos\theta \\ \quad -(S-K-L-M)\delta\cos\iota \end{array} \&c.}{S}$$

$$y = \mathfrak{G}t + \frac{\mathfrak{g}\begin{array}{l} +Kb\cos\zeta - (S-K-L)c\cos\eta \\ \quad -(S-K-L-M)d\cos\theta \\ +K\beta\cos\eta - (S-K-L)\gamma\cos\theta \\ \quad -(S-K-L-M)\delta\cos\iota \end{array} \&c.}{S}$$

$$z = \mathfrak{G}t + \frac{\mathfrak{g}\begin{array}{l} +Kb\cos\zeta + (K+L)c\cos\eta \\ \quad -(S-K-L-M)d\cos\theta \\ +K\beta\cos\eta + (K+L)\gamma\cos\theta \\ \quad -(S-K-L-M)\delta\cos\iota \end{array} \&c.}{S}$$

$$v = \mathfrak{G}t + g \frac{\begin{array}{c} + K b \cos\zeta + (K+L) c \cos\eta + (K+L+M) \\ d \cos\theta \\ + K c \cos\eta + (K+L) \gamma \cos\theta + (K+L+M) \\ \delta \cos\iota \end{array} \&c.}{S}$$

&c.

Deinde ex æquationibus, quas ſollicitationes punctorum K, L, M, N &c. ſuppeditaverunt, colligitur fore.

$$\frac{2K(dpddp + dxddx) + 2L(dqddq + dyddy) + 2M(drddr + dzddz) \ \&c.}{dt^2}$$

$$= \mathfrak{B}(dp - dq) + \mathfrak{C}(dq - dr) + \mathfrak{D}(dr - ds) + \&c.$$

$$+ B(dx - dy) + C(dy - dz) + D(dz - dv) + \&c. \quad =$$

$$- \mathfrak{B} b d\zeta \cos\zeta - \mathfrak{B} c d\eta \cos\eta - \mathfrak{C} \gamma d\theta \cos\theta - \mathfrak{D} \delta d\iota \cos\iota - \&c.$$

$$- \mathfrak{C} c d\eta \cos\eta - \mathfrak{D} d d\theta \cos\theta$$

$$+ B b d\zeta \sin\zeta + B c d\eta \sin\eta + C \gamma d\theta \sin\theta + D \delta d\iota \sin\iota + \&c.$$

$$+ C c d\eta \sin\eta + D d d\theta \sin\theta$$

At ſollicitationes motus rotatorii dabunt

$$\frac{2Kk^2 d\zeta dd\zeta + 2Ll^2 d\eta dd\eta + 2Mm^2 d\theta dd\theta + \&c.}{dt^2} =$$

𝔅

$$\mathfrak{B}\,b\,d\zeta\cos\zeta+\mathfrak{B}\,\epsilon\,d\eta\cos\eta+\mathfrak{C}\,\gamma\,d\theta\cos\theta+\mathfrak{D}\,\delta\,d\iota\cos\iota$$
$$+\mathfrak{C}\,\varepsilon\,d\eta\cos\eta+\mathfrak{D}\,d\,d\theta\cos\theta \qquad \&c.$$
$$-B\,b\,d\zeta\sin\zeta-B\,\epsilon\,d\eta\sin\eta-C\,\gamma\,d\theta\sin\theta-D\,\delta\,d\iota\sin\iota$$
$$-C\,\varepsilon\,d\eta\sin\eta-D\,d\,d\theta\sin\theta$$

quæ expreſſio cum præcedenti ſit æqualis & contrariis ſignis affecta ſequitur fore:

$$\frac{K(dp^2+dx^2+k^2d\zeta^2)+L(dq^2+dy^2+l^2d\eta^2)+M(dr^2+dz^2+m^2d\theta^2)+\&c.}{dt^2}$$

$=$ Conſt. ſicque patet pepetuo eandem virium vivarum quantitatem conſervari.

Quærantur denique valores virium $\mathfrak{B}$, B, $\mathfrak{C}$, C, $\mathfrak{D}$, D, &c. quæ erunt:

$$\tfrac{1}{2}\mathfrak{B}\,dt^2 = K\,ddp$$
$$\tfrac{1}{2}\mathfrak{C}\,dt^2 = K\,ddp + L\,ddq$$
$$\tfrac{1}{2}\mathfrak{D}\,dt^2 = K\,ddp + L\,ddq + M\,ddr$$

&c.

$$\tfrac{1}{2} B\, dt^2 = K\, ddx$$

$$\tfrac{1}{2} C\, dt^2 = K\, ddx + L\, ddy$$

$$\tfrac{1}{2} D\, dt^2 = K\, ddx + L\, ddy + M\, ddz$$

&c.

Si igitur valores pro 𝔅, ℭ, 𝔇, &c. & B, C, D, &c. hinc oriundi in æquationibus ex motibus rotatoriis ortis ſubſtituantur; prodibit

$$K\,kk\,dd\zeta = b(K\,ddp \cos\zeta - L\,ddx \sin\zeta)$$

$$L\,ll\,dd\eta = \begin{cases} - K\,c\,ddp \cos\eta + L\,c\,ddq \cos\eta + K\,\varsigma\,ddp \cos\eta \\ - K\,c\,ddx \sin\eta - L\,c\,ddy \sin\eta - K\,\varsigma\,ddx \sin\eta \end{cases}$$

$$M\,mm\,dd\theta = \begin{cases} - \cos\theta\,(K(\gamma+d)\,ddp + L(\gamma+d)\,ddq + M\,d\,ddr) \\ - \sin\theta\,(K(\gamma+d)\,ddx + L(\gamma+d)\,ddy + M\,d\,ddz) \end{cases}$$

$$N\,nn\,dd\iota = \begin{cases} + \cos\iota\,(K(\delta+\epsilon)\,ddp + L(\delta+\epsilon)\,ddq + M(\delta+\epsilon) \\ \qquad ddr + N\,\epsilon\,dds) \\ - \sin\iota\,(K(\delta+\epsilon)\,ddx + L(\delta+\epsilon)\,ddy + M(\delta+\epsilon) \\ \qquad ddz + N\,\epsilon\,ddv) \end{cases}$$

&c.

Quod

Quod ſi nunc hic loco p, q, r, &c. & x, y, z, &c. valores ſupra inventi ſubſtituantur, habebuntur tot æquationes differentio-differentiales, quot ſunt anguli ζ, η, θ, &c. per easque ergo iſti anguli determinabuntur ex tempore jam elapſo $= t$, quibus inventis ſimul valores coordinatarum p, x, q, y, r, z, &c. innoteſcent, ſicque ad datum tempus poſitio totius corporis una cum motu cujusque articuli definiri poterit.

Q. E. J.

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Tab. I.



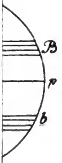




Fig. 9.

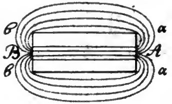
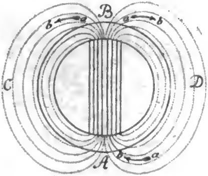


Fig. 8.



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Tab. II.

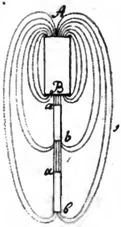
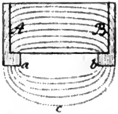


Fig. 17.

Fig. 18.



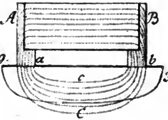


Fig. 19.



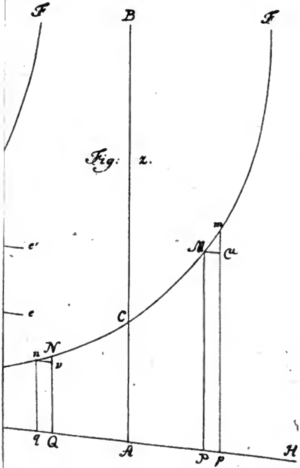
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Tab. III.



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Tab. IV.

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Fig. 2

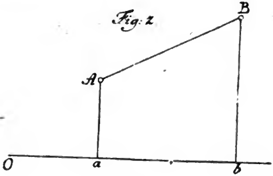


Fig. 5.

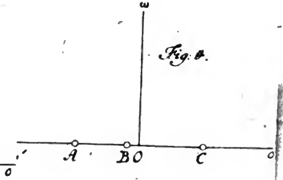
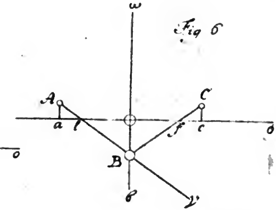


Fig 6

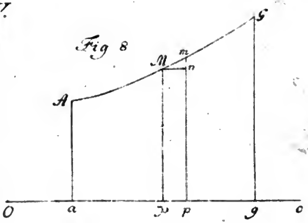


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V.

Fig 8



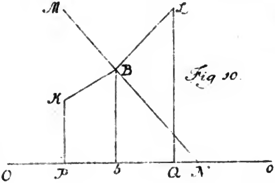
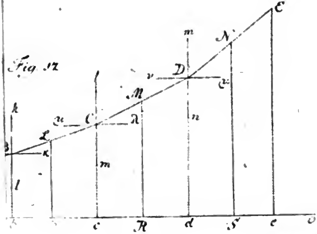


Fig 10.

Fig 12



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